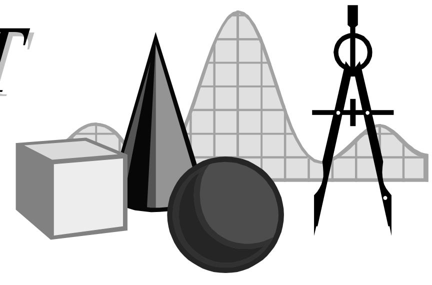
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
Regional Professional Development Program
February 12, 2001 — Middle School Edition



"Choose your language carefully because words mean things."

Whoever first said that might not have been speaking of mathematics, but mathematically it could not be truer. In mathematics (and everyday life) we use words interchangeably when we should not, or we use words with multiple meanings inappropriately. Two such mathematical terms are *inverse* and *opposite*.

First, let us consider the term *inverse* in regard to mathematical operations. The *inverse of an operation* is a second operation that, when performed after the first, "undoes" the first operation. Subtraction is the inverse operation of addition because adding a quantity to a number, then subtracting the same quantity, results in the original number. In the cases of addition/subtraction and multiplication/division, we are speaking of *binary* operations, that is, operations on two objects. An example of a *unary* operation—one number—is $\sqrt{x}$. The inverse of this unary operation is x^2 .

Second, we shall consider the *inverse of an element*. Given some binary operation, a number may have an inverse in that operation. The stipulation is that if the operation is performed on the number and its inverse, the result is the operation's *identity element*. For example, the identity element of addition is 0, because any number plus zero equals the number. Thus, the *additive inverse* of 5 is –5 because 5 + (–5) = 0 and the *multiplicative inverse* of 5 is $\frac{1}{5}$ because $\frac{1}{5} \cdot 5 = 5 \cdot \frac{1}{5} = 1$.

One must keep in mind that not all operations have an identity element. Addition has 0, multiplication has 1, but subtraction and division don't have identity elements. One might suppose that subtraction has an identity element of 0 because any number minus 0 is the number. But, for an operation to have an identity element, the identity must work "commutatively." For example, $5 + 0 = 0 + 5$, but $5 - 0 \neq 0 - 5$. Therefore subtraction does not have an identity element.

Last, we address the term *opposite*. Many texts introduce *opposite* as another name for additive inverse. The opposite of 5 is –5; the opposite of *a* is –*a*. (Remember, these are read "negative five" and "negative '*a*', not "minus five" or "minus '*a*'.") This is an accepted use for the term opposite, but research cannot turn up a book that uses *opposite* with respect to operations. Yet, many teachers say, "The *opposite* of multiplication is division." This is incorrect terminology. We should say, "The *inverse* of multiplication is division."

Few mathematics dictionaries define *opposite* in this manner. (It is usually reserved for geometric concepts.) A few texts and dictionaries do describe additive inverse as "the negative of." For instance, *the negative of* 5 is –5. We must use this phraseology very carefully so we do not give the impression that "negatives of" are always negative. The negative of –3 is 3.