TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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Factoring is a core topic in algebra and one that takes a great deal of practice to master. The most common method for factoring trinomials of the form ax^2+bx+c is *trial-and-error*. With time, coupled with solid number sense, students can usually factor a trinomial without too much effort. Other methods exist that make factoring easier, such as the “bottoms-up” method or the “A-C” method. In this two-page combined issue of *Take It to the MAT*, we’ll examine when we can and can’t use these methods.

“Bottoms-Up” Example 1: Factor $2x^2+7x+6$

Step 1: Multiply the constant term (6) by the coefficient of the quadratic term (2), and drop the quadratic coefficient:	$x^2+7x+12$
Step 2: Factor the resulting trinomial normally:	$(x+4)(x+3)$
Step 3: Rewrite each binomial’s constant term as a fraction, with the constant as the numerator, and the original quadratic coefficient as the denominator:	$\left(x+\frac{4}{2}\right)\left(x+\frac{3}{2}\right)$
Step 4: Simplify the fractions in the constant terms:	$(x+2)\left(x+\frac{3}{2}\right)$
Step 5: In each factor, “move” the denominators of any remaining fraction to be the coefficient of the linear term:	$(x+2)(2x+3)$
The result of Step 5, is the original polynomial in factored form:	$2x^2+7x+6=(x+2)(2x+3)$

“Bottoms-Up” Example 2: Factor $6x^2+2x-20$

Step 1: Multiply the constant term (–20) by the coefficient of the quadratic term (6), and drop the quadratic coefficient:	$x^2+2x-120$
Step 2: Factor the resulting trinomial normally:	$(x+12)(x-10)$
Step 3: Rewrite each binomial’s constant term as a fraction, with the constant as the numerator, and the original quadratic coefficient as the denominator:	$\left(x+\frac{12}{6}\right)\left(x-\frac{10}{6}\right)$
Step 4: Simplify the fractions in the constant terms:	$(x+2)\left(x-\frac{5}{3}\right)$
Step 5: In each factor, “move” the denominators of any remaining fraction to be the coefficient of the linear term:	$(x+2)(3x-5)$
The result of Step 5, is **not** the original polynomial in factored form:	$6x^2+2x-10 \neq (x+2)(3x-5)$

Our factored polynomial actually equals $3x^2+x-10$. Notice that the we have lost a common factor of 2 somewhere. We have several choices here. We could factor out the common factor of two as a first step, $6x^2+2x-10=2\left(3x^2+x-5\right)$, or we could search for a common factor after the fact. The former choice is better, factor out a common factor first—which is what we usually teach kids to do anyway. With trial and error methods, getting that common factor from the start was not as critical. It could be factored out of one of the binomials later if we didn’t see it from the beginning.

The latter choice, however, is undesirable. It would require adding another step to a procedure that already is a “black box” to most students (and some teachers). Besides, how would a student recognize when and when not to go back and look for a common factor? If the answer were, “look every time,” why didn’t we do it from the start?

A third and *highly undesirable* method would be to see if the numerators and denominators in Step 3 **all** have a common factor—in this case they do, 2. That is the “lost” common factor, so we’ll “tack it on” at the end.

Why is “bottoms up” called a “black box?” Without a solid understanding of *why* it works, the “bottoms-up” method is merely a way to “get the answer.” The result from Step 1 of the process **does not** equal the original polynomial, nor do we have equality between Steps 2 and 3 or Steps 4 and 5. Furthermore, if there is a factor common to all terms and it is not factored out, then the procedure does not work, unless we make some post-factoring “adjustment.” While the “bottoms-up” method works, it is probably not the best method and can break down understanding through its repeated use of steps with non-equivalent expressions.

“A-C” Example 1: Factor $2x^2+7x+6$

Step 1: Multiply the constant term (6) by the coefficient of the quadratic term (2):	12		
Step 2: Find two factors of Step 1 that have a sum equal to the linear coefficient (7):	3, 4		
Step 3: Split the linear term into two terms, using the values from Step 2 as coefficients:	$2x^2+4x+3x+6$		
Step 4: Group the terms in pairs and factor out common factors: (Factor by grouping.)	$\left(2x^2+4x\right)+\left(3x+6\right)$ $=2x\left(x+2\right)+3\left(x+2\right)$	**OR**	$\left(2x^2+3x\right)+\left(4x+6\right)$ $=x\left(2x+3\right)+2\left(2x+3\right)$
Step 5: Factor out the common group as you would a common monomial factor:	$\left(x+2\right)\left(2x+3\right)$	**OR**	$\left(2x+3\right)\left(x+2\right)$
The result of Step 5, is the original polynomial in factored form:	$2x^2+7x+6=\left(x+2\right)\left(2x+3\right)$		

“A-C” Example 2: Factor $6x^2+2x-20$

Step 1: Multiply the constant term (–20) by the coefficient of the quadratic term (6):	–120		
Step 2: Find two factors of Step 1 that have a sum equal to the linear coefficient (2):	–10, 12		
Step 3: Split the linear term into two terms, using the values from Step 2 as coefficients:	$6x^2-10x+12x-20$		
Step 4: Group the terms in pairs and factor out common factors: (Factor by grouping.)	$\left(6x^2-10x\right)+\left(12x-20\right)$ $=2x\left(3x-5\right)+4\left(3x-5\right)$	**OR**	$\left(6x^2+12x\right)+\left(-10x-20\right)$ $=6x\left(x+2\right)-10\left(x+2\right)$
Step 5: Factor out the common group as you would a common monomial factor:	$\left(3x-5\right)\left(2x+4\right)$	**OR**	$\left(x+2\right)\left(6x-10\right)$
The result of Step 5, is the original polynomial in factored form:	$6x^2+2x-20=\left(3x-5\right)\left(2x+4\right)=\left(x+2\right)\left(6x-10\right)$		

In the second example our factored polynomial is correct, however, we could still factor 2 out of the second group (as we could have from the original polynomial). Also, it did not matter how we chose to group the terms in Step 4. The point is that the “A-C” method seems to work with polynomials with common factors, unlike “bottoms-up.” We don’t “lose” a common factor if we miss it in the beginning. And the expressions are equivalent all the way through the procedure—no magic from the black box.

The bottom line: There are many methods of factoring trinomials of the form ax^2+bx+c and all lead to the answer. Some, like “bottoms-up,” use some algebraic slight-of-hand. Others, like “A-C,” maintain equivalent expressions throughout the process and are less like magic. The method we use with our students should not only lead to the desired result, a factored polynomial, but also maintain—and not contradict—those algebraic principles they previously learned.