TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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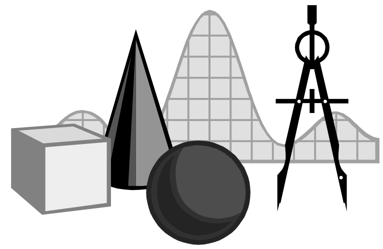

Solving equations is usually the third major topic taught in algebra courses after order of operations and evaluating expressions. Very often, students see equation solving as a purely mechanical process, based on a rigid structure of rules. Equation-solving procedures are seen merely as a way to "get the answer," with little meaning attached. In this issue of *Take It to the MAT*, we will examine ways to make the process more meaningful.

When first learning to solve equations, we often present simple equations such as $x + 4 = 9$. As students have been doing "missing addend" exercises since the primary grades, many can immediately identify the value of x as 5. Others may require the prompt, "What do I add to 4 to get a sum of 9?" Yet, we teachers often begin by teaching them a procedure.

The teacher tells students, "Since four is added to x, subtract 4 from both sides. The 'fours' cancel, so x is 5." The student is taught to write what is shown at right, and we require students to write the step. This "show everything" requirement is one reason that students find equation solving to be little other than a series of answer-finding steps with no real significance.

$$x + 4 = 9$$
$$x + 4 - 4 = 9 - 4$$
$$x = 5$$

In order that students find meaning in equation solving, as well as building their mental math skills, the teaching of equation solving should begin with a reasoning-based approach that is tied to order of operations. After all, the method to solve equations is simply the reverse of order of operations.

Rather than starting with trivial equations, such as $x + 4 = 9$, begin with something where the answer is not obvious: $7x - 19 = 72$, for example. If we had a value for x and wished to verify if it were a solution, we would multiply it by seven, then subtract 19 and see if the result is 72. That's where we want equation solving to begin. Now, have the students reason through it.

A student's thoughts: *"I take seven times something, subtract 19, and that equals 72. Whatever that seven times x is, when I take away 19 I get 72. (Covers 7x with finger.) Take away 19 from what's under my finger to get 72. It must be* ***larger*** *than 72. So what's 72* ***plus*** *19? It's 91. What's under my finger equals 91. (Lifts finger.) So, seven times x is 91. Seven times what is 91? Gotta be* ***smaller*** *than 91. Hmmm…91* ***divided*** *by 7 is 13. So 7 times 13 is 91. OK, x is 13! Let's check…7 times 13 is 91, 91 minus 19 is 72. It works!"*

The trick is to now get the students to go deeper into that thought process, to see that the first step was to *add* 19 to 72, then *divide* by 7. These are the inverse operations in the opposite order than would be used to evaluate the left side of the equation for a given value of x. Linking this to the *properties of equality*—i.e. if $a = b$, then $a - c = b - c$, etc.—starts to give some meaning to the equation solving procedures we teach. Students then have conceptual understanding of the techniques we use to solve equations.

It is not that we shouldn't require students to show their work when solving equations, particularly beyond one step. That should not be inferred from this letter. What we should require is students understand the concept behind the equation-solving process, see its connections to order of operations and equality properties, and be able to mentally solve simple equations quickly and efficiently using some number sense. If students aren't thinking like the student two paragraphs earlier, we must help them—teach them—to think that way.