TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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One of the skills that is taught toward the end of the first year of algebra is simplifying radicals and radical expressions. In this month's *Take It to the MAT*, Eric Johnson of Harney MS in Las Vegas shares a strategy for doing this that connects to the previously learned skill of prime factorization.

Let's start with a simple example: Simplify $\sqrt{18}$. We would normally tell students to look for perfect squares that are factors of 18. That is, they should eventually find that 9 is a square factor of 18, so $\sqrt{18} = \sqrt{9 \cdot 2}$. Remembering that for non-negative real numbers *a* and *b*, $\sqrt{ab} = \sqrt{a}\sqrt{b}$, the expression can be simplified as $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$.

A different approach is to do a prime factorization of the radicand. That is, $\sqrt{18} = \sqrt{2 \cdot 3 \cdot 3}$. We know that $\sqrt{2 \cdot 3 \cdot 3} = \sqrt{2}\sqrt{3}\sqrt{3}$ by the rule cited above. We also know for $a \geq 0$ that $\sqrt{a^2} = \sqrt{a \cdot a} = \sqrt{a}\sqrt{a} = \left(\sqrt{a}\right)^2 = a$. So $\sqrt{2 \cdot 3 \cdot 3} = \sqrt{2}\sqrt{3}\sqrt{3} = \sqrt{2} \cdot 3 = 3\sqrt{2}$. Perhaps a better way to look at this is that we can "pull out" any pair of like factors from the radicand, since the product of that pair is a perfect square.

Let's look at a little longer, yet simple, example. Simplify $\sqrt{72}$. The steps are shown in the box at right. There are two factors of 2 and two factors of 3 in the radicand, so we "pull them out." Now, we would hope that once students extracted that first factor of 2 and saw that 72 = 2 · 36, they would know that 36 = 6 · 6 and "pull out" the six. But we'll assume they didn't for the sake of the procedure here, because its usefulness becomes more apparent with higher indexed roots.

$$\sqrt{72}$$
$$= \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$
$$= 2 \cdot 3\sqrt{2}$$
$$= 6\sqrt{2}$$

If we were to find cube roots, then we would need to "pull out" like factors in triplets, fourth roots would require quadruplets, and so forth. The index of the root tells what size the groups need to be that we "pull out." If we asked students to simplify $\sqrt[3]{500}$ using this method, they would come up with something like what is shown at right.

$$\sqrt[3]{500}$$
$$= \sqrt[3]{2 \cdot 2 \cdot 5 \cdot 5 \cdot 5}$$
$$= 5\sqrt[3]{4}$$

This can even be extended to monomial expressions containing variables. For instance, simplify $\sqrt[4]{80x^6}$, for $x \geq 0$. The result is again shown in the box at right.

$$\sqrt[4]{80x^6}$$
$$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$$
$$= 2x\sqrt[4]{5x^2}$$

Don't we want kids to be able to simplify radicals by recognizing squared and cubed factors? Of course we do, and we should expect as much! While the method presented here seems laborious, and does not initially appear to meet that goal, it does reinforce the concept that $\sqrt[n]{a^n} = a$, for $a \geq 0$. It also won't be long before kids start memorizing cubes, fourths, and other powers of 2, 3, 5, etc. to save steps. They'll also skip breaking a power of a variable into single factors and will just immediately "pull out" the variable in groups equal to the index.