



TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



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One of the major misconceptions in algebra is the meaning of a variable. Quite often, students do not have a solid understanding of what a variable represents. Consider the following question:

At tutoring club, there are three students for every tutor. This can be represented by the equation $s = 3t$. Interpret the meaning of the variable t .

How many students would write, “ $t = \text{tutors?}$ ” How many of us would?

This is, of course, an incorrect answer. The correct interpretation of the variable t is “ $t = \text{the number of tutors.}$ ” A student merely writing “tutors” is unacceptable, but understandable. A teacher writing it is sloppy, and is deepening students’ misconceptions about the meaning of a variable. (The author stands guilty of exhibiting such sloppiness on more than one occasion and awaits pronouncement of sentence.)

A variable, whether representing something that can indeed vary, like in the question above, or a fixed unknown, as in $21 = 3t$, is a number—a quantity. This must be taught, reinforced, practiced, and assessed.

It is well documented that students will interpret a variable as being a one letter abbreviation for a word or name. In the case of the question above, it is not unusual for students first working with variables to read $3t$ as “three tutors” instead of “three times *the number of tutors.*” This is understandable, since they have been working with measurement since the earliest grades. A two-liter bottle of soda is labeled 2 L. A room 10 meters wide is noted as 10 m. Why would $3t$ be any different?

It’s all about context. When we give a measurement, the letter or letters after the numeral indicate a unit of measure and is/are an abbreviation. A 9-volt battery is labeled as 9 V. Completing a task in 13 seconds is shown as 13 s. Legal size paper is 14 in. long. Even when we have a single unit—a meter stick is 1 m long—we include the abbreviation.

On the other hand, when we describe unknown or variable quantities, the letter after the numeral, if there is a numeral, may be a physical measurement or it may not. The key is that it is a quantity, not a thing, an object, or a unit of measure. The *number* of cases of soda purchased for a party may be represented by c ; the number of cans of soda is therefore $24c$, and the cost (in dollars) might be written as $8.49c$. In this context, when we ask how many cans were purchased, the expression $24c$ does not mean “24 cases” but “24 times *the number of cases.*” Even the expression $1c$, which we typically write without the coefficient, means “one times *the number of cases*” not “one case.”

If we hammer, drive home, drill, push, emphasize—pick your verb—the fact that variables represent *quantities*, not individuals, things, or units of measure, our students will do better in writing expressions, solving equations, and solving problems. It’s important that we teachers do not get lazy or sloppy when we are using and describing variables.