TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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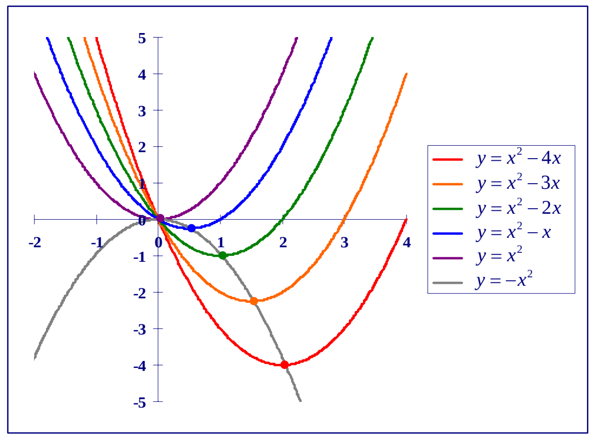
In the study of parabolas defined in the general form $y = ax^2 + bx + c$, we are all familiar with what happens as we change the values of a and c. Increase the magnitude of a, and the parabola gets relatively narrower than before. Decrease a's magnitude and the parabola becomes wider. Changing the value of c translates the parabola vertically on the coordinate plane. What's most intriguing, however, is what happens when we vary the value of b. In this edition of *Take It to the MAT*, we'll examine just that.

First, we'll start with a base parabola, $y = x^2$. (Shown in violet.) There's no point in complicating matters by having a equal anything other than 1 or c be a value other than 0.



Now look at what we get when $-4 \le b \le 0$. As b decreases, the parabolas tend to shift both rightward and downward. What's more interesting is that in tracing the pattern of the vertices it is not linear.

Examine the coordinates of the vertices for the values of b given in the table. The x-coordinates $\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$ follow a linear pattern. This should not be surprising to us since the x-coordinate of the vertex is $h = -\dfrac{b}{2a}$, so we get the linear function with respect to b, $h = -\dfrac{1}{2}b$

b	vertex (h, k)
0	$(0, 0)$
-1	$(\frac{1}{2}, -\frac{1}{4})$
-2	$(1, -1)$
-3	$(\frac{3}{2}, -\frac{9}{4})$
-4	$(2, -4)$

The y-coordinates are more fascinating: $\{0, -\frac{1}{4}, -1, -\frac{9}{4}, -4\}$. Actually, it might be better to write them as $\{\frac{0}{4}, -\frac{1}{4}, -\frac{4}{4}, -\frac{9}{4}, -\frac{16}{4}\}$. This pattern is quadratic, where $k = -\dfrac{1}{4}b^2$. Again, we should not be stunned by this fact, since we are drawing parabolas of the form $y = x^2 + bx$. Thus, $k = h^2 + bh = \left(-\dfrac{1}{2}b\right)^2 + b\left(-\dfrac{1}{2}b\right) = -\dfrac{1}{4}b^2$.

We have established that quadratics of the form $y = x^2 + bx$ have $h = -\dfrac{1}{2}b$ and $k = -\dfrac{1}{4}b^2$. We'll eliminate the parameter b by first solving $h = -\dfrac{1}{2}b$ for b to get $b = -2h$. Substituting that into $k = -\dfrac{1}{4}b^2$ we find $k = -\dfrac{1}{4}(-2h)^2 = -h^2$. Remember that the x- and y-coordinates of the vertex are h and k, respectively, so as the parameter b is changed, the vertex traces out the path $y = -x^2$ (shown above in gray).

The derivation of the curve defined by the vertices of parabolas in the general form $y = ax^2 + bx + c$ as b varies is left to the reader (for now).