



TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



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In this issue of *Take It to the MAT*, we'll look at how the distributive property can be a powerful tool in multiplying fractions and mixed numbers.

First, consider a case using the distributive property with multiplication of whole numbers. If we ask students to find the product 4×27 , we should expect them to do this mentally. One way is to consider that 27 is $20 + 7$, so 4×27 is equal to $4 \times (20 + 7)$. Using the distributive property, that's $4 \times 20 + 4 \times 7 = 80 + 28 = 108$. This can all be done in their pointy little heads.

$$\begin{aligned} 4 \times 27 &= 4 \times (20 + 7) \\ &= 4 \times 20 + 4 \times 7 \\ &= 80 + 28 \\ &= 108 \end{aligned}$$

Now, consider the product $4 \times 2\frac{1}{5}$. Many textbooks (and teachers) will instruct students to change the mixed number $2\frac{1}{5}$ to the improper fraction $\frac{11}{5}$ and multiply $4 \times \frac{11}{5}$. That gives us $\frac{44}{5}$ which simplifies to the mixed number $8\frac{4}{5}$. These steps are also shown in the box at right. That seems to be a lot of steps for something that is not so different from 4×27 .

$$\begin{aligned} 4 \times 2\frac{1}{5} &= 4 \times \frac{11}{5} \\ &= \frac{44}{5} \\ &= 8\frac{4}{5} \end{aligned}$$

Why can't we apply the distributive property to $4 \times 2\frac{1}{5}$? What would be wrong with thinking of this as $4 \times \left(2 + \frac{1}{5}\right)$? Applying the distributive property, $4 \times \left(2 + \frac{1}{5}\right) = 4 \times 2 + 4 \times \frac{1}{5} = 8 + \frac{4}{5} = 8\frac{4}{5}$. Once again, these steps could be done mentally. This is not to say that it would be impossible in one's head to change $2\frac{1}{5}$ to $\frac{11}{5}$, multiply it by 4, and convert back to a mixed number. But, using the distributive property seems to flow much better.

$$\begin{aligned} 4 \times 2\frac{1}{5} &= 4 \times \left(2 + \frac{1}{5}\right) \\ &= 4 \times 2 + 4 \times \frac{1}{5} \\ &= 8 + \frac{4}{5} \\ &= 8\frac{4}{5} \end{aligned}$$

There will be times—many times—that some work will have to be done with improper fractions using this method. For example, the product $8 \times 3\frac{1}{6}$ will require some conversion on the back end, but it's not that tough. Using the distributive property, $8 \times 3\frac{1}{6} = 24 + \frac{8}{6}$. Students should be able to mentally hold 24 in their head, realize that $\frac{8}{6}$ is $1\frac{2}{6}$, and simplify $\frac{2}{6}$ to $\frac{1}{3}$. So, in the end, we have $8 \times 3\frac{1}{6} = 24 + \frac{8}{6} = 24 + 1 + \frac{1}{3} = 25\frac{1}{3}$.

Pencil-and-paper skills are important, as is knowing how to multiply with mixed numbers by converting them to improper fractions. But so is doing mental computations by using the distributive property.