



# TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



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In this edition of *Take It to the MAT*, we continue our discussion of enhancing estimation skills that began with the October 2005 issue.

At the end of the December 2005 issue, readers were encouraged to think about how to refine such estimates as  $50 \times 20$  for the product  $47 \times 22$ . Our estimate of the product would be 1000. Is that too high or too low? Can we determine this by looking at how much we rounded the numbers? Let's review what we did with addition before answer those questions.

If we were to estimate the sum  $47 + 22 = 69$ —which we wouldn't, but for the sake of an example go along—by using  $50 + 20 = 70$ , it makes sense that our estimate is too high, since we rounded 47 up more than we rounded 22 down. That is, in rounding 47 to 50, we would make our sum 3 too big; in rounding 22 to 20, we would make our sum 2 too small. Thus, our estimate would be 1 larger than the actual sum. (This can be a good mental math strategy for computing sums, by the way.)

Does the same hold true for multiplication? Since we rounded 47 up more than we rounded 22 down, would our estimate of 1000 for  $47 \times 22$  also be too large? It turns out that  $47 \times 22 = 1034$ . Our estimate is actually too small. Why?

When we estimated with addition, we looked at the *absolute differences* between our rounded addends and the actual addends. With multiplication, we do not look at the absolute differences, but the *relative differences* between the rounded factors and the actual factors. That is, what the difference is as a *fraction* of the actual factor.

When we round 22 to 20, there is a difference of 2. Compare 2 to 22—think of it as a fraction:  $\frac{2}{22}$  or

$\frac{1}{11}$ . When 47 is rounded to 50, there is a difference of 3. Compare 3 to 47, and again think of the

fraction  $\frac{3}{47}$  which is about  $\frac{1}{16}$ . Which fraction is larger? That will tell us how our estimate compares with the actual product.

When we rounded 22 down to 20, we made our potential product smaller by a factor of  $\frac{1}{11}$ . When we

rounded 47 to 50, we made our potential product larger by a factor of about  $\frac{1}{16}$ . Since  $\frac{1}{11}$  is larger than

$\frac{1}{16}$ , we rounded down by a greater *fraction* than we rounded up, so our estimate of 1000 will be too small.

If we have numbers that we round the same direction—both up or both down—it's a no-brainer as to whether our estimate will be too large or too small. But if we round opposite directions, then the *relative (fractional) differences* determine the direction of our estimate from the actual product.

Oh, one last thing. A bit of a disclaimer here: this method doesn't *always* work. If one were to randomly make up numbers and apply this technique, we would be wrong about 1–2% of the time. And all those times our estimate is too small. So, be aware that there are rare cases where this strategy for revising estimates sends us the wrong direction, but that should not detract from using it.