TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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What is it about logarithms that kids don't get? Just about everything, it seems. Perhaps we just need to explain things a little differently. In this issue of *Take It to the MAT*, we'll take a brief look at logarithms and their properties.

Let us begin with a textbook definition:

> For any number $b > 0$, $b \neq 1$, $y = \log_b x$ if and only if $x = b^y$.

Nothing overpowering there, but what does it really mean? Here's a statement from a mathematics dictionary that follows the previous definition, but is not often in textbooks:

> "Thus, $\log_b x$ is the power to which the base b must be raised to get the argument x."

Wow! That nails it down—a logarithm is a power, or better yet, *an exponent*! This is the heart of the matter. Our students don't often see that a logarithm is merely an exponent. If they recognized this, they might understand logarithms better. By the time kids reach second-year algebra, they should have a handle on b being the *base* of the exponential expression b^y because it's at the bottom. In the expression $\log_b x$, the base is also easily recognized as b because it's at the bottom. Once the base is established, the rest is a snap.

More specifically, students should quickly evaluate 4^3 to be 64. It's not a great step for them to look at $\log_4 64$, and ask the question, "What power (exponent) of the base 4 gives the argument of 64?" The answer to that is 3, of course, and should be apparent.

Once students master the concept that a logarithm is essentially an exponent, properties of logarithms should come more quickly. We want students to memorize the properties of exponents, to be sure, but they could re-derive them if need be. (It's not a matter of if they'll forget them, but when.)

For instance, take the product rule for exponents $b^m \cdot b^n = b^{m+n}$. Whatever the results of b^m and b^n are, when they are multiplied, the product is equivalent to b^{m+n}. When multiplying expressions with equivalent bases, the exponents add. *The exponents add.* Students should recognize that since logarithms are essentially exponents, there should be times when they add, too.

Let's say that $b^m = x$ and $b^n = y$. Then $\log_b x = m$ and $\log_b y = n$. Since $\log_b x$ and $\log_b y$ are exponents, what would happen if they were added? What is $\log_b x + \log_b y$? Since, $b^m \cdot b^n = b^{m+n}$, then $xy = b^{m+n}$, so the exponent $m + n$ is $\log_b xy$. The sum of the exponents/logarithms is the exponent of the product of the arguments.

Now, expressions like $\log_b 3 + \log_b 9$ are easily done. We're adding logarithms, so we're really adding exponents. When we added exponents with like bases, we were multiplying the arguments—in this case, 3 and 9. Thus, $\log_b 3 + \log_b 9 = \log_b 27$. **Logarithms are exponents!**