TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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Over the last couple of months, *Take It to the MAT* has explored various strategies to compare fractions that employ number sense instead of algorithmic procedures. The most challenging case is when two fractions are on the same side of one half, particularly if they are close together. In this issue, we'll explore a strategy that the author calls the "batting average" or "winning percentage" strategy. It works well for fractions that are clearly close to each other.

Consider the fractions $\frac{8}{21}$ and $\frac{9}{23}$. Both are obviously less than one-half, and (using the "unit fraction" strategy from last month) both are between one-third and one half. Now, think of a baseball player who has 8 hits in 21 at-bats going into a game. The player's batting average as a fraction is $\frac{8}{21}$. During the game, the player gets 1 hit in 2 at-bats, finishing the game with a total of 9 hits in 23 at-bats. We know that before the game, the player's batting average was $\frac{8}{21}$, which was less than half. During the game, the player's average was $\frac{1}{2}$. Since the player hit better during the game than before the game, the player's average must have increased. Thus, $\frac{8}{21} < \frac{9}{23}$.

Here's a second example: compare $\frac{11}{17}$ and $\frac{13}{22}$. Both are greater than one-half, and each numerator is about two-thirds of the denominator. A team entered a tournament having won 11 out of 17 games and finished it having won 13 of 22. That means the team won 2 of 5 games in the tournament or $\frac{2}{5}$. Since $\frac{11}{17}$ is greater than half, but $\frac{2}{5}$ is less than half, the team's winning percentage must have gone down. Therefore, $\frac{11}{17} > \frac{13}{22}$.

This type of thinking connects well to student grades. Students learn fairly quickly that to raise their average in a class, they have to score above that average. (Weighted averages can be a little tricky, but they don't apply in this situation.) If a student has scored 70 points out of a possible 100, thus having an "average" of 70%, the student must score above 70% on his next assignment to raise the overall average. The average will move in the direction of the score of the next assignment.

One note of caution: Students should not infer that this method of comparison also allows arithmetic with the fractions. In the first example, a batter who is 8 for 21 going into a game, and goes 1 for 2 during the game, is now batting 9 for 23. But we *cannot* infer that $\frac{8}{21} + \frac{1}{2} = \frac{9}{23}$. We can only add fractions when the "whole" is the same—there are different wholes in this example. Here, we are combining sets, not adding fractions of the same whole.