TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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Last month, we looked at comparing fractions using number sense. Three situations were covered: like denominators, like numerators, and fractions on opposite sides of one-half. In this issue of *Take It to the MAT*, we'll focus on a fourth case: both fractions on the same side of one-half.

One strategy is to estimate how many times the numerator divides the denominator. This gives an approximate *unit* fraction. For example, we wish to compare $\frac{3}{11}$ and $\frac{5}{28}$. In the first fraction, 3 divides 11 between three and four times. That means that $\frac{1}{4} < \frac{3}{11} < \frac{1}{3}$. In the second fraction, 5 divides 28 a little less than six times. So, $\frac{1}{6} < \frac{5}{28} < \frac{1}{5}$. Since a number between one-fifth and one-sixth is less than a number between one-third and one-fourth, then $\frac{5}{28} < \frac{3}{11}$.

This strategy can be used for numbers greater than one-half by looking at how much is "missing" from the whole. Consider the fractions $\frac{8}{11}$ and $\frac{23}{28}$. For $\frac{8}{11}$, there is $\frac{3}{11}$ missing from the whole. For $\frac{23}{28}$, there is $\frac{5}{28}$ missing from the whole. In the previous example, we saw that $\frac{5}{28} < \frac{3}{11}$, so there is less "missing" from the whole for $\frac{23}{28}$ than $\frac{8}{11}$. Therefore, $\frac{8}{11} < \frac{23}{28}$.

There are times that the fractions are so close, that the unit fraction strategy needs a little enhancement.

Consider the fractions $\frac{8}{21}$ and $\frac{9}{23}$. In the first fraction, 8 divides 21 between two and three times. Likewise, 9 divides 23 between two and three times. Now, the enhancement. If we take $21 \div 8$, we get 2 remainder 5, or better yet $2\frac{5}{8}$. Taking $23 \div 9$, we get 2 remainder 5, or $2\frac{5}{9}$. We can therefore think of the original fractions as $\frac{8}{21} = \frac{1}{2\frac{5}{8}}$ and $\frac{9}{23} = \frac{1}{2\frac{5}{9}}$. Remember, when numerators are equal, larger denominators mean smaller fractions. Since $2\frac{5}{8} > 2\frac{5}{9}$, then $\frac{1}{2\frac{5}{8}} < \frac{1}{2\frac{5}{9}}$, thus $\frac{8}{21} < \frac{9}{23}$.

This last strategy requires a little sophisticated thinking, but builds fraction sense. Students should be encouraged to think of numbers in this way before diving into algorithms. This unit fraction notion will also help them think of relative sizes of fractions—how $\frac{a}{b}$ equates to $\frac{1}{n}$.

Next month, we'll look at *more* number-sense strategies when fractions are on the same side of one-half.