Take It to the MAT

A Newsletter Addressing the Finer Points of Mathematics Instruction

Southern Nevada Regional Professional Development Program
November 2005 — High School Edition

www.rpdp.net

In this issue of *Take It to the MAT*, we're going to start out looking at finding means when observations have different weights, or when we want to find a combined mean of different groups.

Consider this classic problem. *In Class A, the mean score on the last exam was 80 points. In Class B, the mean score on the last exam was 85 points. If there are 30 students in Class A and 20 students in class B, what is the mean score for the two classes combined?* Piece of cake!

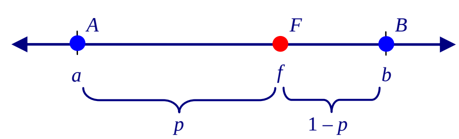
If the mean score in Class A is 80 points, and there are 30 students, then the total number of points scored is 2400. In Class B, the 20 students have a mean of 85 points, so there are a total of 1700 points. The two classes had a combined total of 4100 points among the 50 students. That's a mean of 82 points. A mathematical expression would look like this: combined mean = $\frac{(30)(80)+(20)(85)}{50}=\frac{4100}{50}=82$.

If the two classes were the same size, then the mean would be 82.5, halfway in between the two class means. Since the larger class had the lower class mean (80), the overall mean is pulled in that direction—less than 82.5.

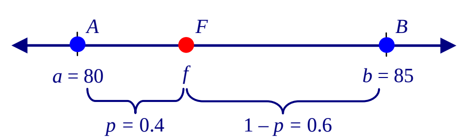
This is the same situation as a single student scoring 80% on a 30-point test and an 85% on a 20-point test. The student's overall mean is just the weighted mean of the two test scores: overall mean = $\frac{30}{50}(80)+\frac{20}{50}(85)=48+34=82$. Once again, the 30-point test carries more weight, so the overall mean will be on the 80% side of 82.5%.

Wait a minute! This looks familiar. Have we seen this before?

In the last issue of *Take It to the MAT*, we looked at a new way to locate a point some fraction of the distance between two others. Recall, the point F is located at f, the fraction p of the way between points A and B. (See diagram.) We found that $f=(1-p)a+pb$. This is the same situation we have above.



Consider the two classes. The larger class (A) carries more weight than the smaller (B), so the weighted mean will be pulled toward A's mean, away from the midpoint. Class B has only 20 out of the 50 students, or 40%, so the weighted mean will be 40% of the way from A to B. If we call the weighted mean f, the means of Classes A and B are a and b, respectively, and Class B carries a weight of p, then $f=(1-p)a+pb=\left(1-\frac{20}{50}\right)(80)+\left(\frac{20}{50}\right)(85)=48+34=82$.



So, the moral of the story is that finding the location of a point a fraction p between two others is merely a weighted mean of the locations of the two points, with the value of p being the weight of the second point!