TAKE IT TO THE MAT

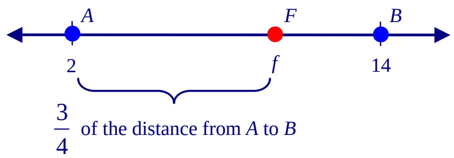
A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Southern Nevada Regional Professional Development Program
October 2005 — High School Edition

www.rpdp.net

In this issue of *Take It to the MAT*, the first of its seventh year, we'll look at situations where we need to find the location of a point some fraction of the way between two other points. By the end of this letter, we'll discover a new and interesting way to locate such points.

Let us begin with a fairly simple example: Points *A* and *B* are located on a number line at values 2 and 14, respectively. If Point *F* is three-fourths of the way from *A* to *B*, what is the value of Point *F* on the number line?



This is fairly easy when we think about it. *F* is located to the right of *A*, three-fourths of the distance between *A* and *B*. We'll label that location *f*. The distance from 2 to 14 is 12, $\frac{3}{4}$ of that distance is 9, so *F* is located 9 units to the right of *A*, or at 11. For the sake of being complete, we'll write it as

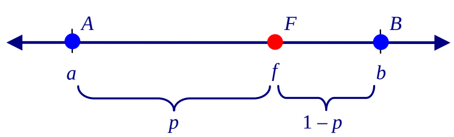
$$f = 2+\frac{3}{4}(14-2) = 2+\frac{3}{4}(12) = 2+9 = 11.$$

Equation 1

$$\begin{aligned} f &= 2+\frac{3}{4}(14-2) \\ &= 2+\frac{3}{4}(14)-\frac{3}{4}(2) \\ &= 2-\frac{3}{4}(2)+\frac{3}{4}(14) \\ &= \left(1-\frac{3}{4}\right)(2)+\frac{3}{4}(14) \\ &= \frac{1}{4}(2)+\frac{3}{4}(14) \end{aligned}$$

Now that we have completed that fairly simple exercise, let's look at the equation above in a little different way. See the box containing *Equation 1*. The **last** line is very interesting. We have the locations of Points *A* and *B* multiplied by complementary fractions—two fractions whose sum is one. What's more intriguing is that those fractions are the respective proportions of $\overline{AB}$ opposite *F*. That is, the location of Point *A* (which is 2) is multiplied by $\frac{FB}{AB}$ $\left(\text{which is } \frac{1}{4}\right)$, and the location of *B* (which is 14) is multiplied by $\frac{AF}{AB}$ $\left(\text{which is } \frac{3}{4}\right)$. Is that just a coincidence? A more general approach will reveal the answer to that question.

On the number line at right, let the locations of Points *A*, *B*, and *F* be *a*, *b*, and *f*, respectively. Let $p=\frac{AF}{AB}$, so $1-p=\frac{FB}{AB}$. The value of *f* is derived in the box *Equation 2*.



Equation 2

$$\begin{aligned} f &= a+p(b-a) \\ &= a+pb-pa \\ &= a-pa+pb \\ &= (1-p)a+pb \end{aligned}$$

So, the point *F* is located at a value equal to the sum of the products of the locations of each endpoint and the fraction of the segment opposite *F* with respect to the endpoint. More succinctly, *F* is located at $f=(1-p)a+pb$. What this provides is an alternate procedure to find the location of a point between two others. (If the case is in two dimensions, each dimension is handled separately, as it is with the traditional method.)

Well, so what? That's a neat trick, but does it mean anything? This is a good question that the reader is invited to think about that until next month's issue.