TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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In this first issue of *Take It to the MAT*'s seventh year, we will look at the skills of comparing and ordering fractions. Procedures to compare fractions are typically introduced in the sixth grade. While these algorithms generate correct answers when executed properly, they are often misunderstood or overused. For a good portion of the narrative ahead, we will look at approaches that are more efficient than algorithms and require students to use number sense.

(The January 16, 2001 Middle School Edition of *Take It to the MAT, Methods of Comparing Fractions*, has more details on algorithmic procedures for comparing fractions and *why* they work.)

For the sake of simplicity, we will confine our discussion to common fractions, those with values between 0 and 1. When comparing two fractions, the pairs can fall into several camps.

1. **Like denominators**. Comparing fractions with like denominators is trivial. Students with a minimal understanding of the concept of fraction should be able to complete the statement $\frac{9}{15}$ ● $\frac{11}{15}$ by filling in ● with the correct =, <, or > symbol.

2. **Like numerators**. Again, comparing fractions with like numerators should be a snap. The understanding that "smaller" denominators mean that each equally-sized part of the whole is larger should be enough to compare $\frac{3}{4}$ and $\frac{3}{5}$, and know that $\frac{3}{4}$ is larger.

3. **One fraction greater than and one fraction less than (or equal to) one-half**. When students are asked to compare $\frac{2}{5}$ and $\frac{5}{8}$, it should not be difficult for them to decide that $\frac{5}{8}$ is greater. One-half is an important benchmark fraction and is integral to fraction sense. In $\frac{2}{5}$ we have two of five equally sized pieces; two is less than half of five so $\frac{2}{5} < \frac{1}{2}$. Similarly, five is more than half of eight, so $\frac{1}{2} < \frac{5}{8}$. By the transitive property, $\frac{2}{5} < \frac{5}{8}$. A quick comparison of fractions to $\frac{1}{2}$ *before* jumping to an algorithm can save a lot of time and reduces the chance of mistakes due to incorrect arithmetic.

A glaring example of where algorithms can hinder students was on a recent standardized test. One question asked one to choose the largest fraction from the following list: $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{12}$, $\frac{5}{7}$, and $\frac{7}{15}$. Diligent test-takers plugged and chugged away finding a common denominator for 4, 7, 8, 12, and 15, writing equivalent fractions, and then comparing the numerators. Studying the fractions carefully, though, reveals that $\frac{5}{7}$ is the only fraction greater than $\frac{1}{2}$ and is therefore the largest.

There is a fourth situation we must consider: **both fractions greater than or less than one-half.** We'll explore that case in more detail next month. But until then, does this letter imply that procedures for comparing fractions should not be taught? Absolutely not! But a little number sense can go a long way.