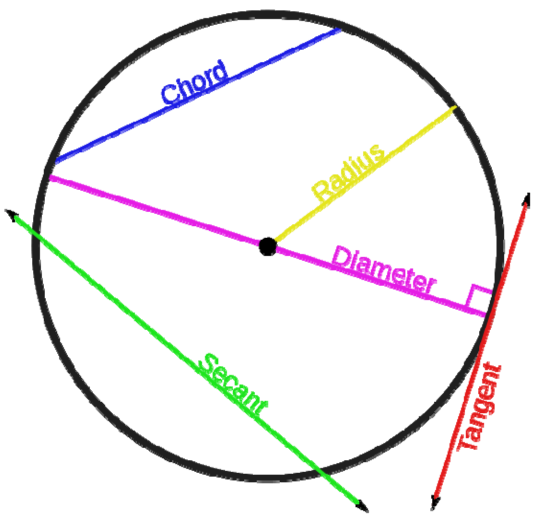
Geometry Unit 10 – Notes
Circles

Syllabus Objective: 10.1 - The student will differentiate among the terms relating to a circle.

Circle – the set of all points in a plane that are equidistant from a given point, called the *center*.



Radius – the distance from the center to a point on the circle.

Diameter – the distance across a circle, through the center. The diameter is twice the radius.

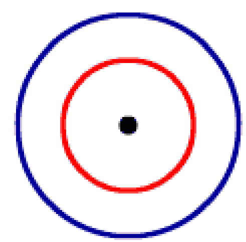
Chord – a segment whose endpoints are points on the circle.

Secant – a line that intersects a circle in two points.

Tangent – a line in the plane of a circle that intersects the circle in exactly one point.

Concentric circles – coplanar circles that have a common center.

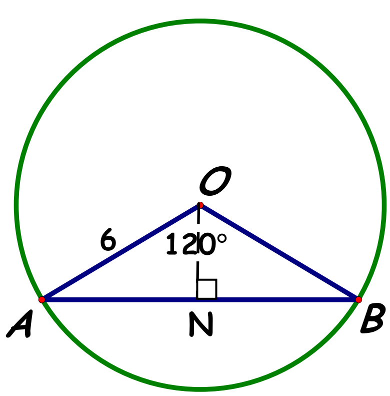
Some real life examples are targets and Target™!



Point of tangency – the point at which the tangent line intersects the circle.

Syllabus Objective: 10.4 - The student will explore relationships among circles and external lines or rays.

Example: Find the length of chord $\overline{AB}$.

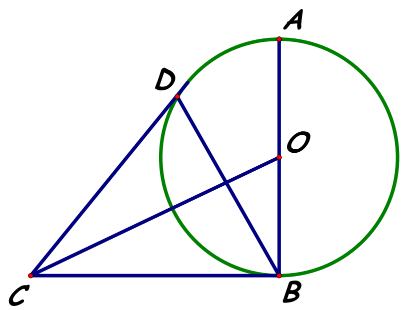


$OA = OB = 6$.
Draw $\overline{ON} \perp \overline{AB}$. $\overline{ON}$ bisects $\overline{AB}$;
$\overrightarrow{ON}$ bisects $\angle AOB$. Using
the properties of $30 \circ 60 \circ 90$
triangles, $ON = 3$ and
$AN = 3\sqrt{3}$, so
$AB = 2\left(3\sqrt{3}\right) = 6\sqrt{3}$.

Theorem: If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Theorem: In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

Example: $\overleftrightarrow{CB}$ **and** $\overleftrightarrow{CD}$ **are tangent to** $\odot O$ **at B and D, respectively.**



If $AB = 12$ and $BC = 8$, then $OC =$ ___?___.

Since AB passes through the center, it is perpendicular to the tangent line $\overline{BC}$. The diameter is 12, so the radius $\left(\overline{OB}\right)$ is 6. Use the Pythagorean Theorem to find OC.

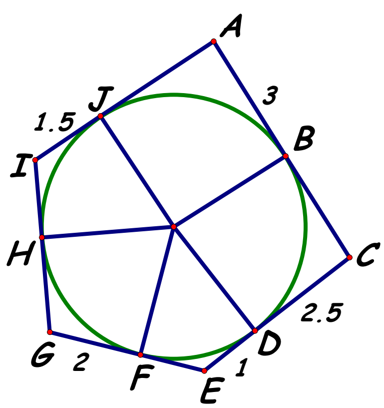
$8^2 + 6^2 = OC^2$,

$OC = 10$.

Theorem: If two segments from the same exterior point are tangent to a circle, then they are congruent. {Think of an ice cream cone, the two sides should be equal where they meet the ice cream.}

Example:

a) Find the perimeter of the polygon.



$AB = AJ = 3$,

$CB = CD = 2.5$,

$ED = EF = 1$,

$GF = GH = 2$,

$IH = IJ = 1.5$.

Add them all together, $P = 20$ units.

Syllabus Objective: 10.3 - The student will solve problems involving arcs, chords, and radii of a circle.

<u>Central angle</u> – an angle whose vertex is the center of a circle.

<u>Minor arc</u> – part of a circle that measures less than 180°.

<u>Major arc</u> – part of a circle that measures between 180° and 360°.

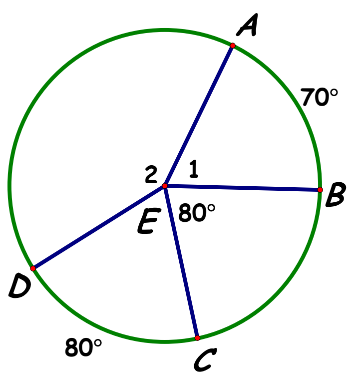
Semicircle – an arc whose endpoints are the endpoints of a diameter of the circle. A semicircle measures exactly 180°.

Arc Addition Postulate: The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Theorem: In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

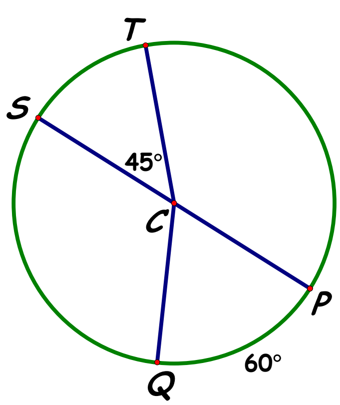
Examples:

a) In $\odot E$, find the measure of the angle or the arc named.



	Solutions:
$\overparen{BC}$	$m\overparen{BC} = m\angle BEC = 80°$.
$\angle 1$	$m\angle 1 = m\overparen{AB} = 70°$.
$\overparen{AC}$	$m\overparen{AC} = m\overparen{AB} + m\overparen{BC}$ $= 70 + 80 = 150°$.
$\overparen{ADB}$	$m\overparen{ADB} = 360 - m\overparen{AB}$ $= 360 - 70 = 290°$.

b) In $\odot C$ with diameter $\overline{SP}$, find the measure of the angle or the arc named.



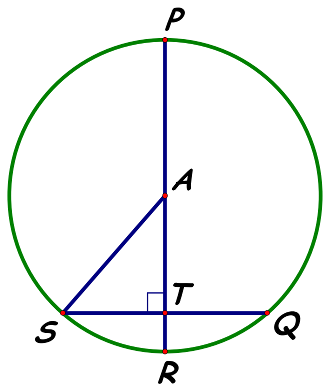
	Solutions:		**Solutions:**
$\angle PCQ$	60°	$\overparen{SPQ}$	240°
$\overparen{ST}$	45°	$\overparen{PT}$	135°
$\overparen{SQP}$	180°	$\angle TCP$	135°
$\overparen{SQ}$	120°	$\overparen{SPT}$	315°
$\angle SCQ$	120°	$\overparen{TSQ}$	165°
$\angle SCP$	180°		

Theorem: If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

Theorem: If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

Examples:

a) In $\odot A$, $SQ = 12$ and $AT = 8$. Find PR.



$ST = TQ = \frac{1}{2}SQ = \frac{1}{2}(12) = 6,$

$(SA)^2 = (ST)^2 + (AT)^2,$

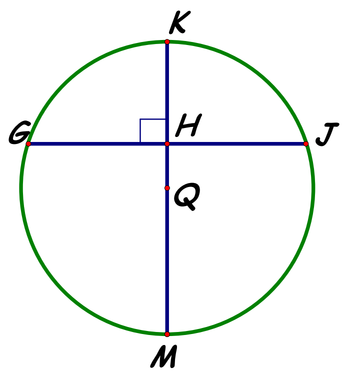
$= 6^2 + 8^2,$

ΔATS is a 6-8-10 right triangle, so

$SA = 10.$

$\overline{PR}$ is a diameter, so $PR = 2(SA) = 20.$

b) $m\widehat{GMJ} = 200°$, $m\widehat{GK} =$ ___?___.



Since $\overline{KM}$ is perpendicular to $\overline{GJ}$,

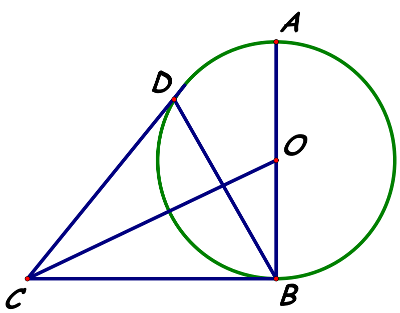
it also bisects $\overline{GJ}$ and $\widehat{GJ}$.

$360 - 200 = 160°,$

$m\widehat{GJ} = 160°,$

$m\widehat{GK} = \frac{1}{2}m\widehat{GJ} = \frac{1}{2}(160) = 80°.$

c) $\overleftrightarrow{CB}$ and $\overleftrightarrow{CD}$ are tangent to $\odot O$ at B and D, respectively., if $m\angle BCD = 50°$, then $m\angle DBO =$ ___?



ΔBCD must measure 180°.

$180 - 50 = 130°$ to be shared evenly by

$\angle BDC$ and $\angle DBC$.

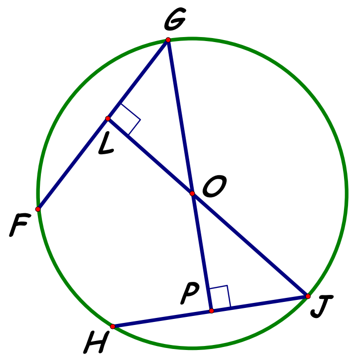
$m\angle DBC = 65°$, $m\angle OBC = 90°$. $90 - 65 = 25°$.

$m\angle DBO = 25°.$

Theorem: In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

<u>Example:</u>

In $\odot O$, $FL = 3$, $GO = 5$, $OP = 4$. Find HJ.



L is the midpoint of $\overline{FG}$, so $LG = 3$.

In rt. ΔOLG, $5^2 = 3^2 + (OL)^2$.

Then $25 - 9 = 16 = (OL)^2$, $OL = 4$.

Since $OP = 4$, $OL = OP$ and $\overline{FG} \cong \overline{HJ}$,

$HJ = FG = 2FL = 6$.

Syllabus Objective: 10.2 - The student will solve problems involving angles, arcs, or sectors of circles.

One easy way for students to remember the angle measures relationships: have them imagine a rubber band being held down on two points of the circle. As the rubber band is stretched to the center of the circle it will form an angle, which is equal to the arc measure. Stretched some more and it will be thinner (one-half the sum of the arcs). Stretched even further, to the edge of the circle and it will be thinner (one-half the arc). And stretched further again, even thinner still (one-half the difference of the arcs).

<u>Inscribed angle</u> – an angle whose vertex is on a circle and whose sides contain chords of the circle.

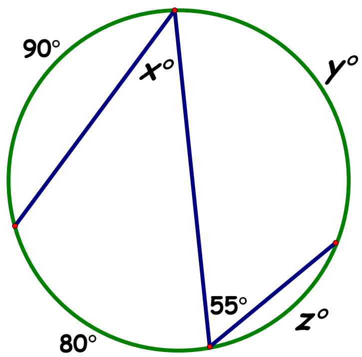
<u>Intercepted arc</u> – the arc that lies in the interior of an inscribed angle and has endpoints on the angle.

<u>Inscribed polygon</u> – drawn inside. A polygon is inscribed in a circle if all its vertices lie on the circle.

<u>Circumscribed</u> – drawn around. In the above definition, the circle is circumscribed about the polygon.

Measure of an Inscribed Angle Theorem: If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc. $\{angle = \frac{1}{2} arc\}$

Example: Find the values of x, y, and z.



$x = \frac{1}{2}(80) = 40,$

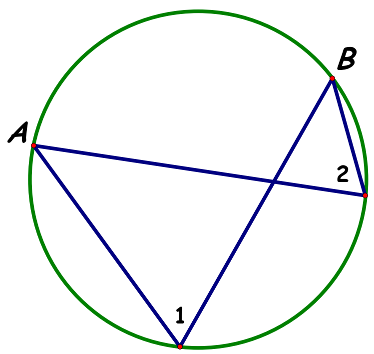
$\frac{1}{2}y = 55,\ y = 110,$

$z = 360 - (80 + 90 + 110) = 80.$

Theorem: If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

Example:

If $m\overset{\frown}{AB} = 120$, then $m\angle 1 =$ __?__ and $m\angle 2 =$ __?__.



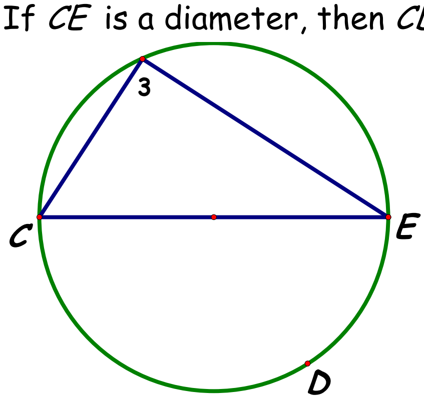
Since $\angle 1$ and $\angle 2$ both intercept $\overset{\frown}{AB}$.

$m\angle 1 = m\angle 2 = \frac{1}{2}m\overset{\frown}{AB} = 60°.$

Theorem: If a right triangle is inscribed is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle. $\{angle = \frac{1}{2}arc\}$

Example:

If $\overline{CE}$ is a diameter, then $\overset{\frown}{CDE}$ is a __?__ and $m\angle 3 =$ __?__.

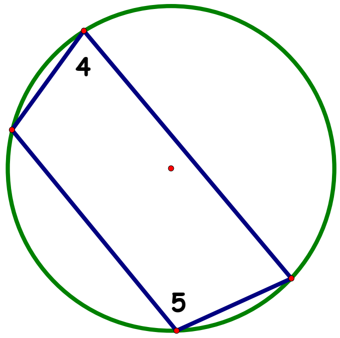


$\overset{\frown}{CDE}$ is a semicircle, so $m\angle 3 = 90°$.

Theorem: A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

Example:

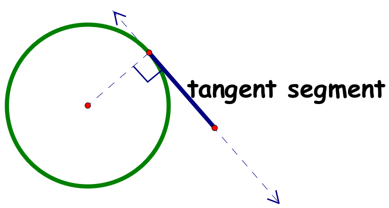
If $m\angle 4 = 75^\circ$, then $m\angle 5 =$ __?__.



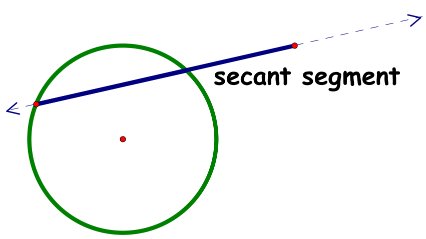
Since opposite angles of an inscribed quadrilateral are supplementary,
$m\angle 5 = 180 - 75 = 105^\circ$.

Syllabus Objective: 10.6 - The student will solve problems involving secant segments and tangent segments for a circle.

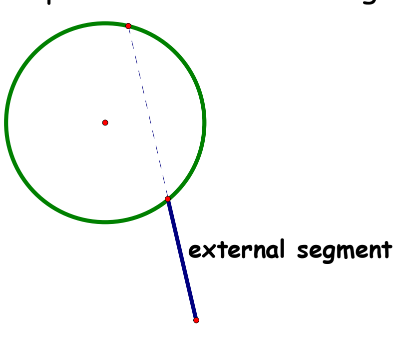
Tangent segment – a segment that is tangent to a circle at an endpoint.



Secant segment – a segment that intersects a circle in two points, with one point as an endpoint of the segment.



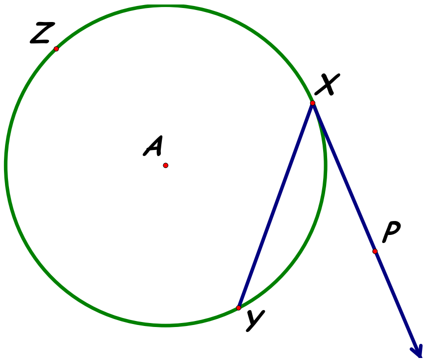
External segment – the part of the secant segment that is not inside the circle.



Theorem: If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc. $\{angle = \frac{1}{2} arc\}$

Example:

If $\overrightarrow{XP}$ is tangent to $\odot A$ and $m\overset{\frown}{XZY} = 260^\circ$, find $m\overset{\frown}{XY}$ and $m\angle PXY$.



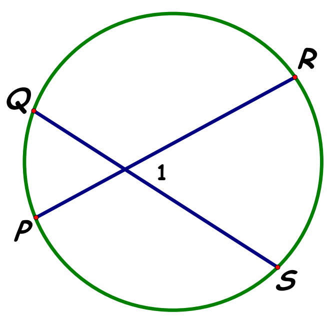
$$m\overset{\frown}{XY} = 360 - m\overset{\frown}{XZY}$$
$$= 360 - 260 = 100^\circ.$$
$$m\angle PXY = \frac{1}{2}m\overset{\frown}{XY}$$
$$= \frac{1}{2}(100) = 50^\circ.$$

Theorem: If two chords intersect in the interior of a circle, then the measure of each angle is one half the <u>sum</u> of the measures of the arcs intercepted by the angle and its vertical angle.

$\left\{ angle = \frac{1}{2}(arc + arc) \right\}$

Examples:

a) If $m\overset{\frown}{PQ} = 45^\circ$ and $m\overset{\frown}{RS} = 75^\circ$, find $m\angle 1$.



$$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{RS} + m\overset{\frown}{PQ}\right)$$
$$= \frac{1}{2}(75 + 45) = 60^\circ.$$

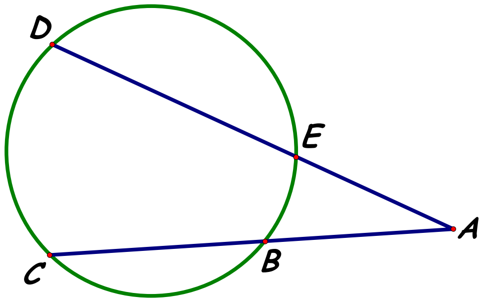
b) If $m\angle 1 = 55^\circ$ and $m\overset{\frown}{RS} = 80^\circ$, find $m\overset{\frown}{PQ}$.

$$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{RS} + m\overset{\frown}{PQ}\right)$$
$$55 = \frac{1}{2}\left(80 + m\overset{\frown}{PQ}\right)$$
$$110 = 80 + m\overset{\frown}{PQ}$$
$$30^\circ = m\overset{\frown}{PQ}.$$

Theorem: If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is one half the <u>difference</u> of the measure of the intercepted arcs. $\left\{ angle = \frac{1}{2}(\text{larger } arc - \text{ smaller } arc) \right\}$

Examples:

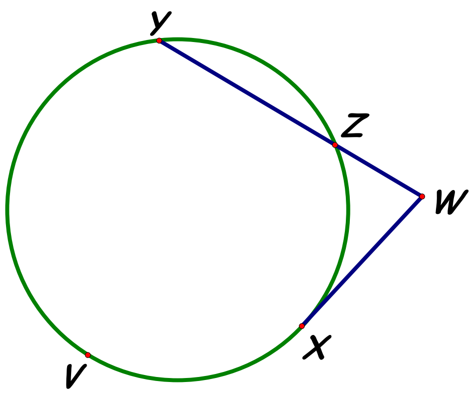
a) If $m\overset{\frown}{DC} = 100°$ and $m\overset{\frown}{EB} = 40°$, find $m\angle A$.



Secant-Secant

$$m\angle A = \frac{1}{2}\left(m\overset{\frown}{DC} - m\overset{\frown}{EB}\right)$$
$$= \frac{1}{2}(100 - 40)$$
$$= 30°$$

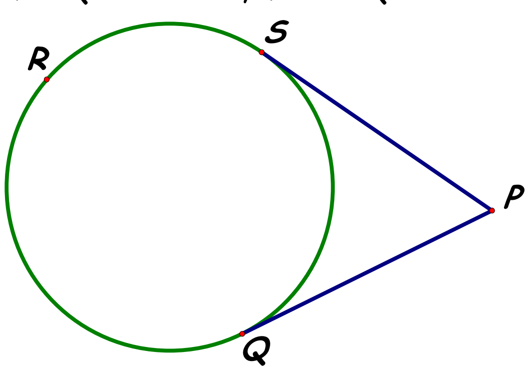
b) If $m\angle W = 65°$ and $m\overset{\frown}{XZ} = 70°$, find $m\overset{\frown}{XVY}$.



Secant-Tangent

$$m\angle W = \frac{1}{2}\left(m\overset{\frown}{XVY} - m\overset{\frown}{XZ}\right)$$
$$65 = \frac{1}{2}\left(m\overset{\frown}{XVY} - 70\right)$$
$$130 = m\overset{\frown}{XVY} - 70$$
$$200° = m\overset{\frown}{XVY}.$$

c) If $m\overset{\frown}{QRS} = 240°$, find $m\overset{\frown}{QS}$ and $m\angle P$.

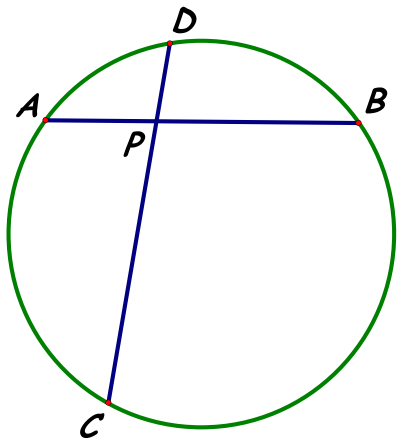


Tangent-Tangent

$$m\overset{\frown}{QRS} + m\overset{\frown}{QS} = 360°,$$
$$240 + m\overset{\frown}{QS} = 360°,$$
$$m\overset{\frown}{QS} = 120°.$$
$$m\angle P = \frac{1}{2}\left(m\overset{\frown}{QRS} - m\overset{\frown}{QS}\right),$$
$$= \frac{1}{2}(240 - 120) = 60°.$$

Theorem: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the products of the lengths of the segments of the other chord.

Examples:



a) If $AP = 4$, $PB = 6$, and $CP = 8$, find PD.

$$AP \bullet PB = CP \bullet PD$$
$$4(6) = 8(PD)$$
$$3 = PD.$$

b) If $AP = 4$, $PB = 9$, and $CD = 15$, find CP.

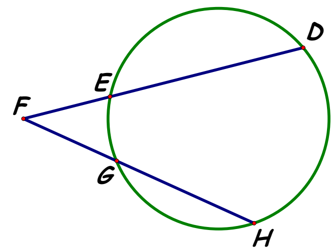
Let $CP = x$; then $PD = 15 - x$.

$$AP \bullet PB = CP \bullet PD$$
$$4(9) = x(15 - x)$$
$$36 = 15x - x^2$$
$$x^2 - 15x + 36 = 0$$
$$(x - 12)(x - 3) = 0$$
$$x = 12 \text{ or } x = 3$$
$$CP = 12 \text{ or } CP = 3.$$

Theorem: If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.
{(entire length)(outside length)=(entire length)(outside length)}

For both of the external segment theorems, students must remember;
(entire length)(part outside) = (entire length)(part outside).
This works for the tangent segments as well because the segment length is also the external segment length (tangent squared).

Examples:



a) If $DF = 17$, $EF = 3$, and $GF = 6$, then $HF =$ __?__.

$DF \cdot EF = HF \cdot GF$,

$(17)(3) = (HF)(6)$,

$HF = 8.5$.

b) If $HG = 8$, $GF = 7$, and $DF = 21$, then $EF =$ __?__.

$DF \cdot EF = HF \cdot GF$,

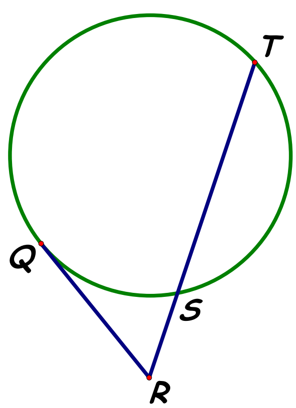
$(21)(EF) = (8+7)(7)$,

$EF = 5$.

Theorem: If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.

{(entire length)(outside length)=(entire length)(outside length)}

Examples:



a) If $RT = 25$ and $RS = 5$, find QR.

{(entire length)(outside length)=(entire length)(outside length)}

$RT \cdot RS = QR \cdot QR$ or $RT \cdot RS = QR^2$,

$(25)(5) = (QR)^2$,

$5\sqrt{5} = QR$.

b) If $QR = 6$ and $ST = 9$, find RS.

Let $RS = x$; then $RT = x + 9$,

$RT \cdot RS = (QR)^2$,

$(x+9)(9) = 6^2$,

$x^2 + 9x - 36 = 0$,

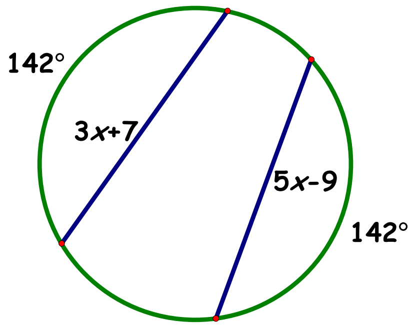
$(x+12)(x-3) = 0$,

$x = -12$ or $x = 3$, so $RS = 3$.

Syllabus Objective: 10.5 - The student will solve problems involving properties circles using algebraic techniques.

Examples:

a) Find the value of x.



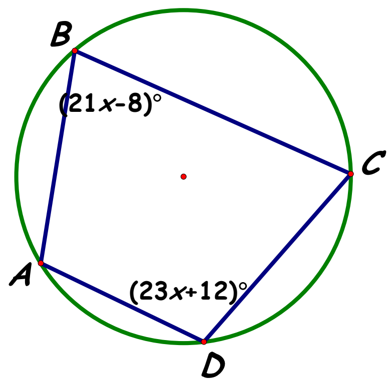
Since the arcs are equal (congruent), the chords are congruent.

$3x+7=5x-9,$

$16=2x,$

$x=8.$

b) Find $m\angle D$ and $m\angle B$.



Since $ABCD$ is inscribed in a circle, opposite angles are supplementary.

$m\angle D+m\angle B=180,$

$23x+12+21x-8=180,$

$44x+4=180,$

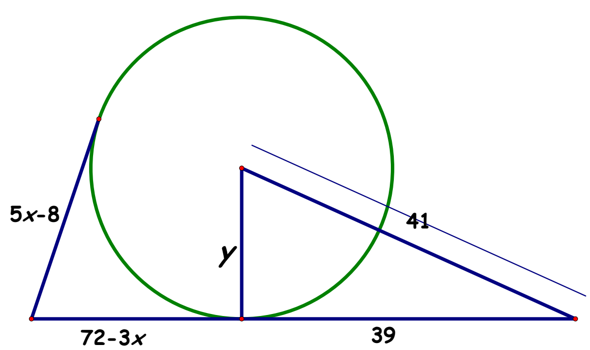
$44x=176,$

$x=4.$

So, $m\angle D=23(4)+12=104°$ and

$m\angle B=21(4)-8=76°.$

c) Find x and y. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.



Two tangent segments are equal so,

$5x-8=72-3x,$

$8x=80,$

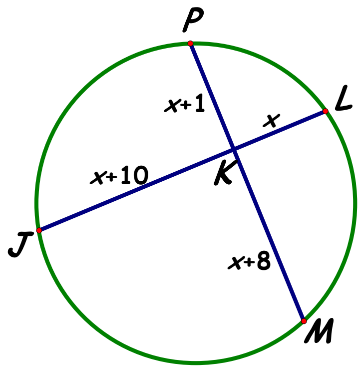
$x=10.$

Radius is perpendicular to tangent line so,

$y^2+39^2=41^2,$

$y=\sqrt{160}=4\sqrt{10}.$

d) Find x.



$JK \bullet KL = PK \bullet KM$

$(x+10)(x) = (x+1)(x+8)$

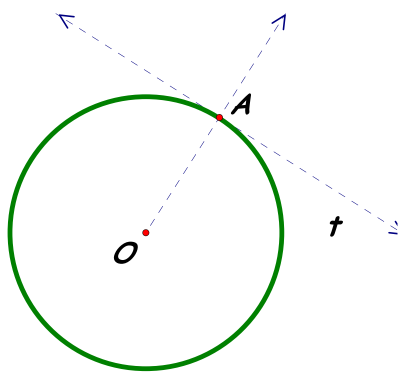
$x^2 + 10x = x^2 + 9x + 8$

$10x = 9x + 8$

$x = 8.$

Syllabus Objective: 10.7 - The student will perform constructions involving special relationships within circles. {May require supplemental material}

<u>Example:</u> <u>Construct a tangent to a given circle from a point on the circle.</u>



Given: Point A on $\odot O$.

Step 1: Draw $\overrightarrow{OA}$.

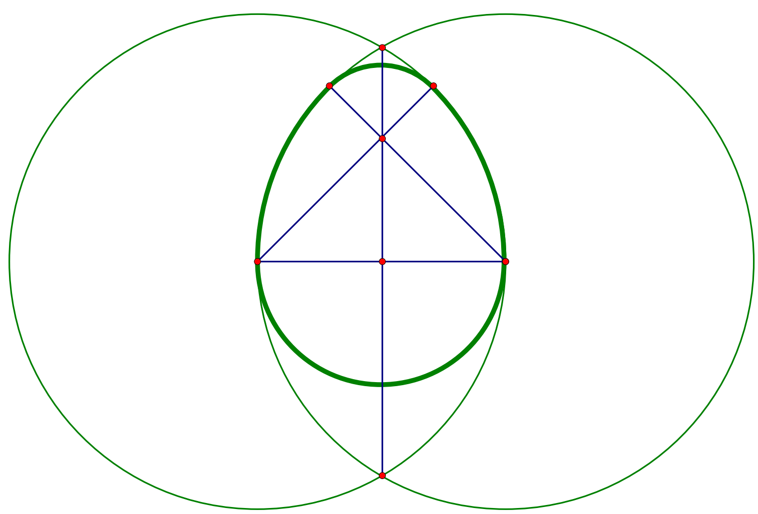
Step 2: Construct a perpendicular from a point ON the line, named t.

Line t is tangent to $\odot O$ at A.

Eggs

Give students the construction steps and see if they can recreate the shape.

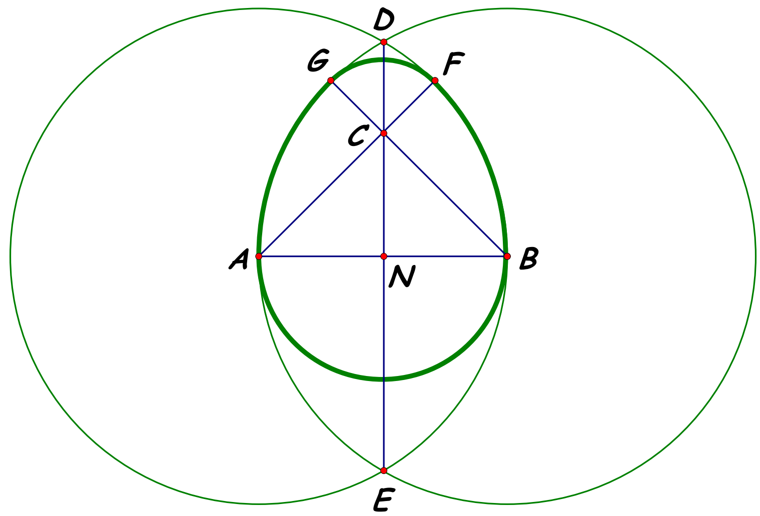
The beautifully smooth egg shown below is composed of circular segments that fit together 'continuously', so that there is no sudden change in the 'direction' of the shell, where any two segments touch.



The Problem :

Reconstruct the egg yourself, using only a compass and a straightedge.

Notes :



The order of construction would be:

1. Draw two congruent circles, the second having its center on the circumference of the first.
2. Join their centers. ($\overline{AB}$)
3. Join their points of intersection. ($\overline{DE}$ – the perpendicular bisector of $\overline{AB}$)
4. Mark the intersection of $\overline{AB}$ and $\overline{DE}$. (the mid-point of $\overline{AB}$ as N)
5. Mark off point C on the line segment $\overline{ND}$, such that $NA = NB = NC$.
6. Lengthen $\overline{AC}$ and $\overline{BC}$ to cut the 'inside' of the original circles at F and G respectively. (and form $\triangle ABC$)
7. Use compasses centered at C, radius $\overline{CF}$, to draw minor circular $\overset{\frown}{FG}$.
8. Use compasses centered at N, radius $\overline{NB}$, to draw semi-circular $\overset{\frown}{AB}$.
9. Go over $\overset{\frown}{FG}$, $\overset{\frown}{GB}$, $\overset{\frown}{BA}$ and $\overset{\frown}{AF}$ to emphasize the required "Egg" shape.

Syllabus Objective: 10.8 - The student will graph a circle and determine its equation.

<u>Standard equation of a circle</u> – a circle with radius r and center (h, k) has this standard equation: $(x-h)^2+(y-k)^2=r^2$

<u>Examples:</u> **<u>Write the equation of the circle.</u>**

a) Center at $(1,-8)$, radius $\sqrt{7}$.

$$(x-h)^2+(y-k)^2=r^2,$$
$$(x-1)^2+(y-(-8))^2=\left(\sqrt{7}\right)^2,$$
$$(x-1)^2+(y+8)^2=7.$$

b) Center at $(-2,4)$, passes through $(-6,7)$.

Use distance formula to find radius:
center to point on circle.

$$r = \sqrt{(-6-(-2))^2 + (7-4)^2},$$

$$r = \sqrt{(-4)^2 + (3)^2},$$

$$r = \sqrt{16+9} = \sqrt{25} = 5.$$

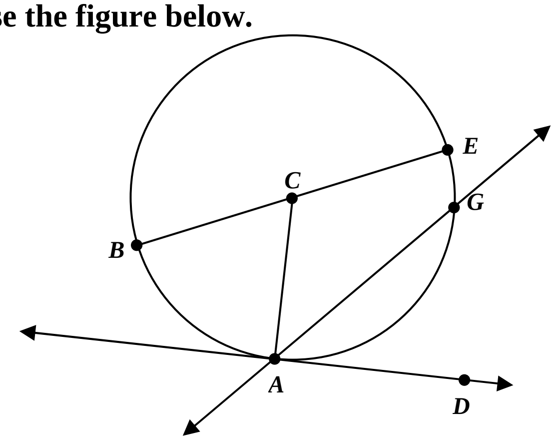
$$(x-h)^2 + (y-k)^2 = r^2,$$

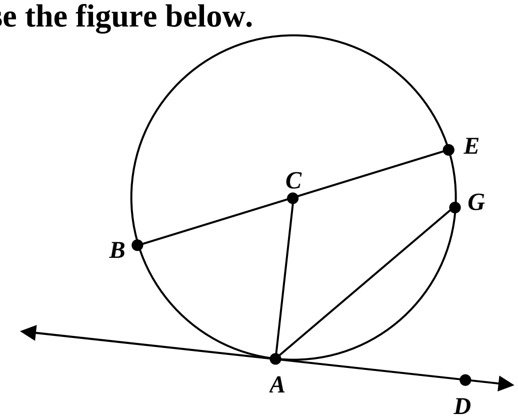
$$(x-(-2))^2 + (y-4)^2 = (5)^2,$$

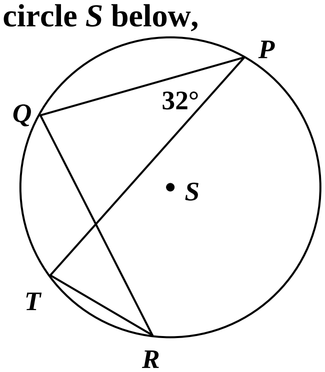
$$(x+2)^2 + (y-4)^2 = 25.$$

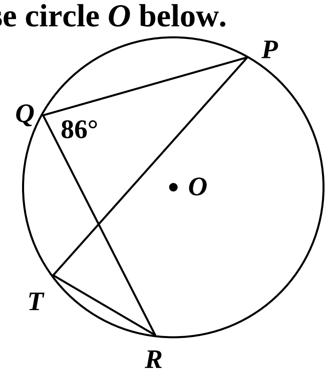
This unit is designed to follow the Nevada State Standards for Geometry, CCSD syllabus and benchmark calendar. It loosely correlates to Chapter 10 of McDougal Littell *Geometry* © 2004, sections 10.1 – 10.6. The following questions were taken from the 2nd semester common assessment practice and operational exams for 2008-2009 and would apply to this unit.

Multiple Choice

<table>
<tr><th>#</th><th>Practice Exam (08-09)</th><th>Operational Exam (08-09)</th></tr>
<tr><td>13.</td><td>Which accurately describes a tangent?
A. A segment whose endpoints are on the circle.
B. A line that intersects a circle in two points and passes through the center of the circle.
C. A segment having an endpoint on the circle and an endpoint at the center of the circle.
D. A line that intersects a circle at exactly one point.</td><td>Which accurately describes a secant?
A. A line that intersects a circle at two points.
B. A segment whose endpoints are on the circle.
C. A segment having an endpoint on the circle and an endpoint at the center of the circle.
D. A line that intersects a circle at exactly one point.</td></tr>
<tr><td>14.</td><td>Use the figure below.


Which of the following represent a secant?
A. $\overleftrightarrow{AG}$
B. $\overline{BE}$
C. $\overline{CA}$
D. $\overleftrightarrow{DA}$</td><td>Use the figure below.


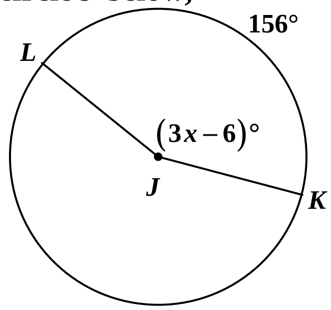
Which represents a chord?
A. $\overline{CA}$
B. $\overline{BC}$
C. $\overline{AD}$
D. $\overline{AG}$</td></tr>
<tr><td>15.</td><td>In circle S below,


The $m\angle QPT = 32°$, what is the measure of $\angle QRT$?
A. 16°
B. 32°
C. 64°
D. 128°</td><td>Use circle O below.


Since $m\angle PQR = 86°$, what is the measure of $\angle PTR$?
A. 43°
B. 86°
C. 90°
D. 172°</td></tr>
</table>

16.

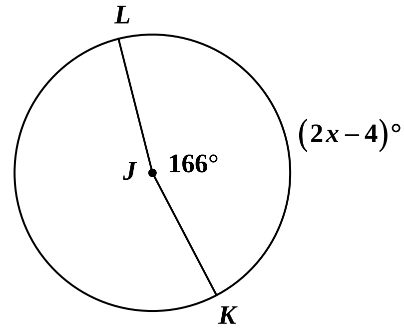
In circle *J* below,



What is the value of *x*?

A. 78
B. 54
C. 50
D. 27

Use circle *J* below.

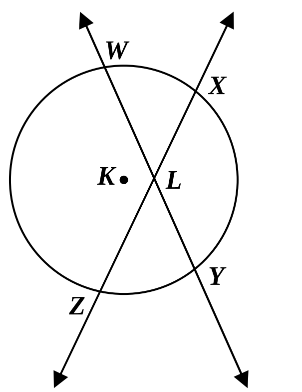


What is the value of *x*?

A. 47
B. 81
C. 85
D. 92

17.

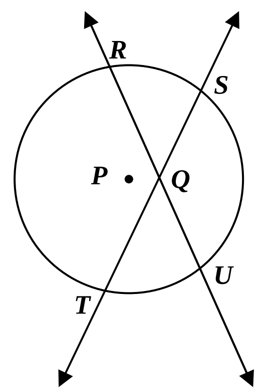
In $\odot K$, $m\overset{\frown}{XY} = (7x-9)°$, $m\overset{\frown}{WZ} = 3(2x+15)°$, and $m\angle XLY = 148°$.



What is the value of *x*?

A. 20
B. 54
C. 131
D. 350

In $\odot P$, $m\overset{\frown}{RS} = (2x+9)°$, $m\overset{\frown}{TU} = (x+7)°$, and $m\angle RQS = 32°$.

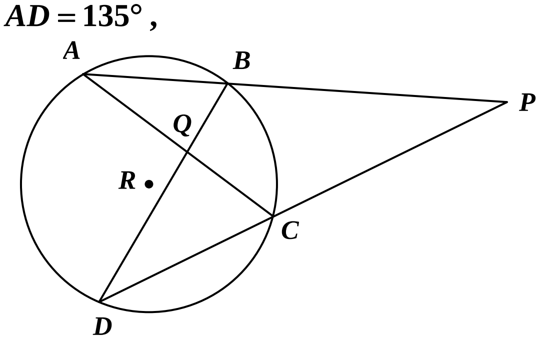


What is the value of *x*?

A. 16
B. 24
C. 48
D. 62

18.

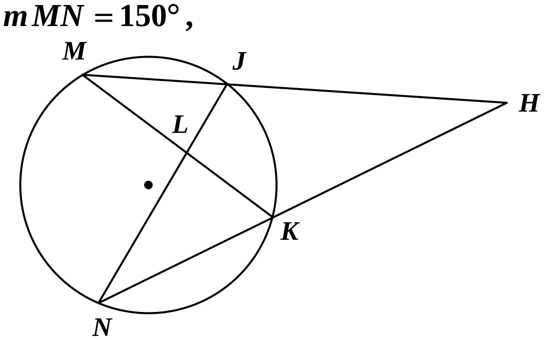
In the figure below, $m\overset{\frown}{BC} = 75°$ and $m\overset{\frown}{AD} = 135°$,



What is $m\angle P$?

A. 15°
B. 30°
C. 45°
D. 60°

In the figure below, $m\angle H = 13°$ and $m\overset{\frown}{MN} = 150°$,

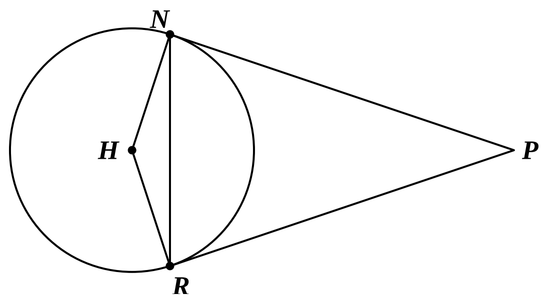


What is the $m\overset{\frown}{JK}$?

A. 88°
B. 101°
C. 124°
D. 176°

19.

Two tangents are drawn from point *P* to circle *H*.



What conclusion is guaranteed by this diagram?

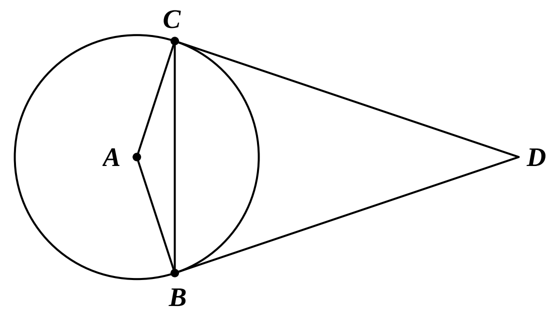
A. $\frac{1}{2} m\widehat{NR} = m\angle NPR$

B. ΔHNR is a right triangle.

C. *HNPR* is a rhombus.

D. *HNPR* is a kite.

Two tangents are drawn from point *D* to circle *A*.



What conclusion is guaranteed by this diagram?

A. $\frac{1}{2} m\widehat{BC} = m\angle BDC$

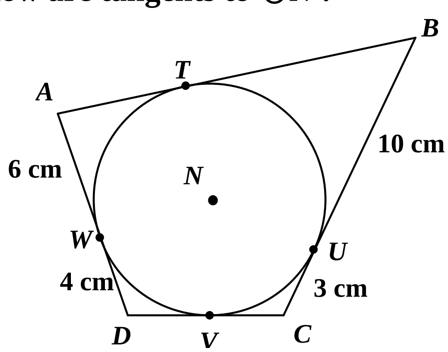
B. ΔABC is a right triangle

C. $BC = BD$

D. ΔBCD is an isosceles triangle

20.

All of the segments shown in the figure below are tangents to $\odot N$.



Given the measures in the figure above, what is the perimeter of quadrilateral *ABCD*?

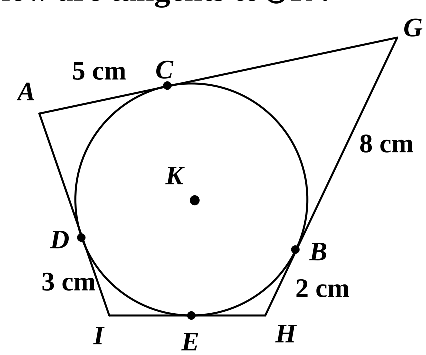
A. 23 cm

B. 40 cm

C. 46 cm

D. 52 cm

All of the segments shown in the figure below are tangents to $\odot K$.



Given the measures in the figure above, what is the perimeter of quadrilateral *AGHI*?

A. 18 cm

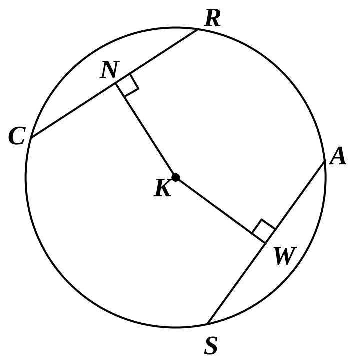
B. 26 cm

C. 31 cm

D. 36 cm

21.

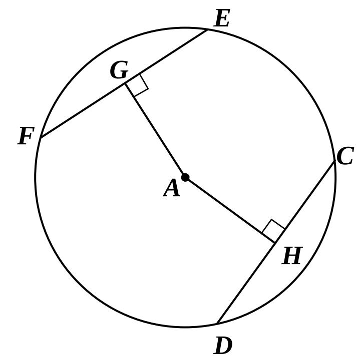
In $\odot K$, $NK = 3x + 4$, $KW = 5x - 8$, $SA = 5x - 4$, and $\overline{KN} \cong \overline{KW}$.



What is CN?

A. 6
B. 13
C. 22
D. 26

In $\odot A$, $\overline{CD} \cong \overline{FE}$, $CH = 4x + 2$, and $FE = 6x + 10$.

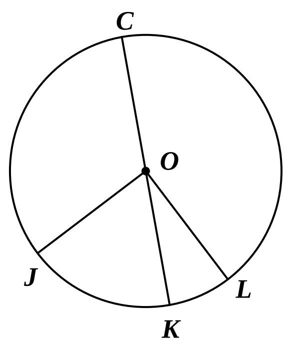


What is the value of x?

A. –4
B. –3
C. 3
D. 4

22.

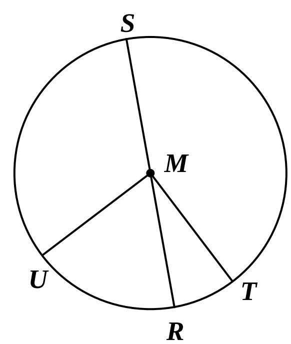
$\overline{CK}$ is the diameter of $\odot O$, $m\widehat{JC} = (19x)°$, and $m\widehat{JK} = (9(x+2) - 6)°$.



What is the value of x?

A. $\frac{4}{5}$
B. $\frac{5}{6}$
C. 4
D. 6

$\overline{SR}$ is the diameter of $\odot M$, $m\angle RMT = (x + 15)°$, $m\angle UMR = (3x + 15)°$, and $m\angle SMT = (4x - 10)°$.

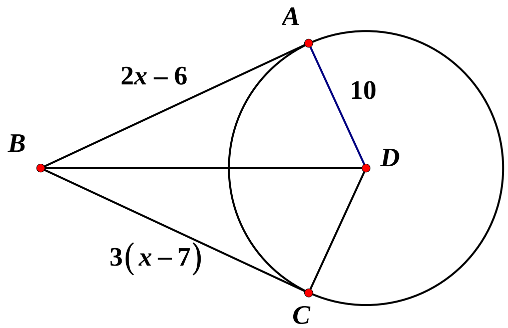


What is the value of x?

A. 35
B. 25
C. 20
D. 5

48.

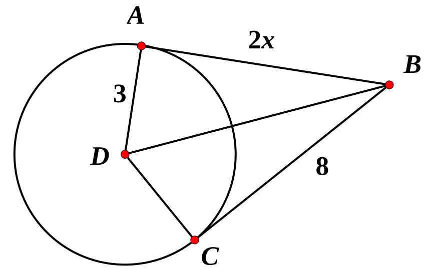
In circle *D* below, $\overline{AB}$ is tangent to $\odot D$ at *A*, and $\overline{CB}$ is tangent to $\odot D$ at *C*.



What is the length of $\overline{BD}$?

A. 14
B. 15
C. 24
D. 26

In the figure below, $\overline{AB}$ is tangent to $\odot D$ at *A* and $\overline{CB}$ is tangent to $\odot D$ at *C*.

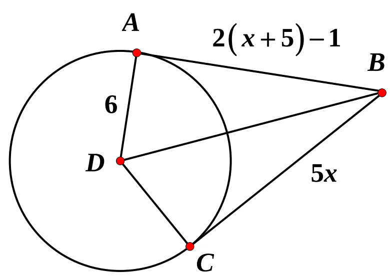


Find the value of *x*.

A. 2
B. 3
C. 4
D. 5

49.

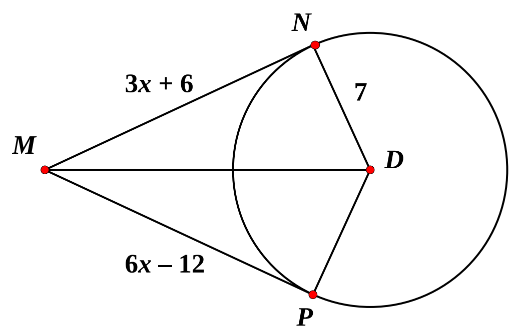
In the figure below, $\overline{AB}$ is tangent to $\odot D$ at *A* and $\overline{BC}$ is tangent to $\odot D$ at *C*.



What is the value of *x*?

A. 2
B. 3
C. 4
D. 5

In circle *D* below, $\overline{MN}$ is tangent to $\odot D$ at *N* and $\overline{MP}$ is tangent to $\odot D$ at *P*.

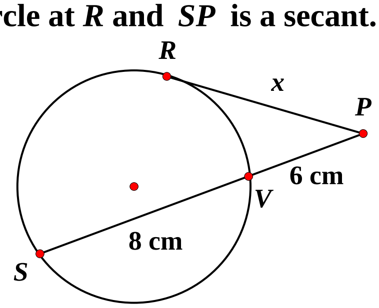


What is the length of $\overline{MP}$?

A. 25
B. 24
C. 7
D. 6

50.

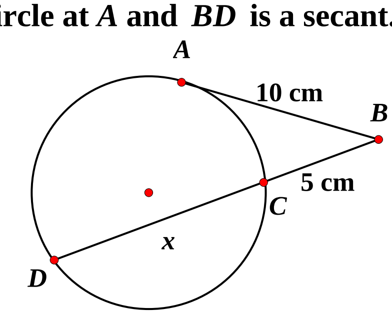
In the figure below, $\overline{RP}$ is tangent to the circle at *R* and $\overline{SP}$ is a secant.



What is the value of *x*?

A. 48 cm
B. 84 cm
C. $4\sqrt{3}$ cm
D. $2\sqrt{21}$ cm

In the figure below, $\overline{AB}$ is tangent to the circle at *A* and $\overline{BD}$ is a secant.



What is the value of *x*?

A. 2 cm
B. 5 cm
C. 15 cm
D. 25 cm