Geometry Unit 9 – Notes
Surface Area and Volume

Review topics: *1)polygon* *2)ratio* *3)area formulas* *4)scale factor*

Polyhedron – a solid that is bounded by polygons, called *faces*, that enclose a single region of space. Plural is *polyhedra*, or *polyhedrons*.

Syllabus Objective: 9.1 - The student will compare attributes of various geometric solids.

Faces – the polygonal regions on a solid bounded by edges.

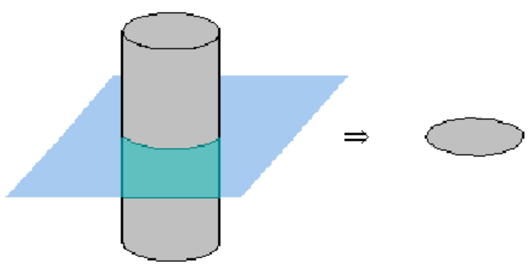
Edges – a line segment formed by the intersection of two faces of a polyhedron.

Vertices – a point where three or more edges of a polyhedron meet.

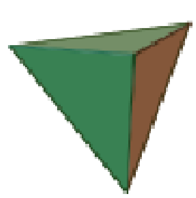
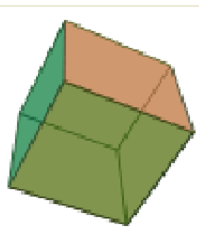
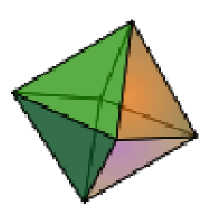
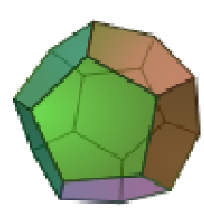
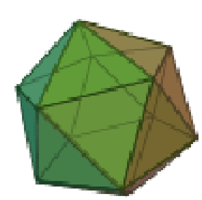
Base(s) – one face (or two congruent faces) of a solid that determine the category of a specific solid.

Cross Section – the intersection of a plane and a solid.

The circle on the right is a cross section of the cylinder on the left.



Platonic Solids – Five regular polyhedra, named after the Greek mathematician Plato, including a regular tetrahedron, a cube (regular hexahedron), a regular octahedron, a regular dodecahedron, and a regular icosahedron.

The Five Platonic Solids				
Tetrahedron	Cube (or Hexahedron)	Octahedron	Dodecahedron	Icosahedron
				

Euler's Theorem: The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F+V=E+2$.

Examples:

a) A polyhedron has 10 vertices and 7 faces. Find the number of edges.
$F+V=E+2 \rightarrow 7+10=E+2 \rightarrow 17=E+2 \rightarrow E=15$ (a pentagonal prism).

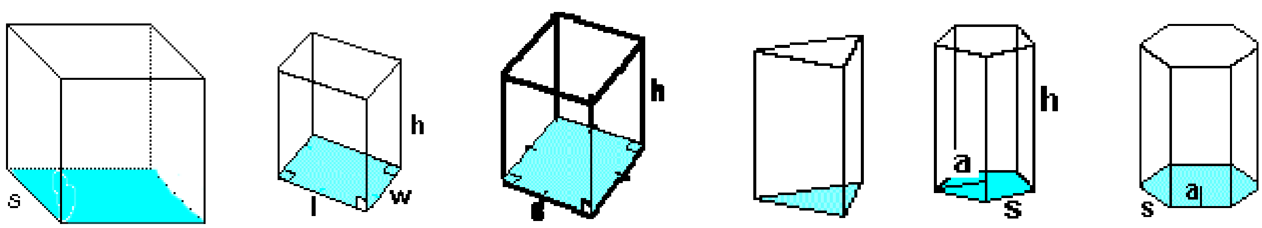
b) Is it possible for a polyhedron to have 6 faces, 6 vertices and 6 edges?
$F+V=E+2 \rightarrow 6+6\ ?\ 6+2 \rightarrow 12 \neq 8 \rightarrow$ NO.

Syllabus Objective: 9.2 - The student will solve surface area and volume problems of various geometric solids.

Review concept: *Special right triangles.*

Many common household items can be used to demonstrate these concepts. Try to collect a few as you go through the year so that you don't have to depend on the students to bring them in.

Prism – a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called *lateral faces*, are parallelograms formed by connecting corresponding vertices of the bases. The segments connecting the vertices are *lateral edges*. The *altitude*, or *height*, of a prism is the perpendicular distance between the bases.



Cube	Rectangular Prism	Regular Square Prism	Triangular Prism	Regular Pentagonal Prism	Regular Hexagonal Prism

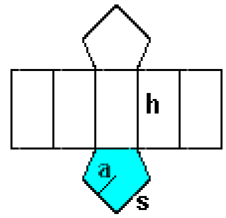
Surface Area – the measure of how much exposed area a solid has. {S}

Lateral Area – the surface area of a solid with the area of the bases (if any) excluded.
{prism or cylinder: $LA=Ph$ or $2\pi rh$, pyramid or cone: $LA=\frac{1}{2}Pl$ or πrl (l is the slant height) }

Make sure students are aware: lateral area and surface area are still area questions. Answers are still in square units.

Net – the two-dimensional representation of all of the faces of a polyhedron. As if the edges were opened and it was laid out flat.

Net of a Regular Pentagonal Prism:



Volume of a solid – the number of cubic units contained in its interior.

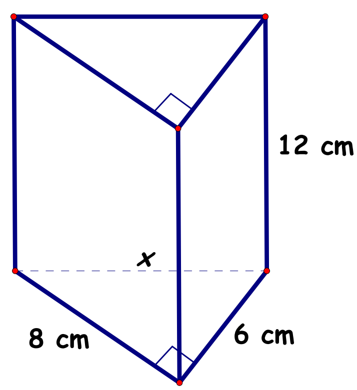
Surface Area of a Right Prism Theorem: The surface area S of a right prism can be found using the formula $S = 2B + Ph$, where B is the area of a base, P is the perimeter of a base, and h is the height. $\{S = 2B + Ph$ or $S = LA + 2B$ and $LA = Ph\}$

A method for finding the Surface Area of a Right Rectangular Prism:
$S = 2lw + 2lh + 2wh$ (length, width, and height)

Remind students that **B** will be replaced with the area formula of the base shape.

Examples:

a) Find the lateral area and the total surface area of the triangular prism.



First find the value of x: $x^2 = 6^2 + 8^2 \rightarrow x = 10$.

Perimeter of base: $P = 6 + 8 + 10 = 24$ cm. Height of prism: $h = 12$ cm.

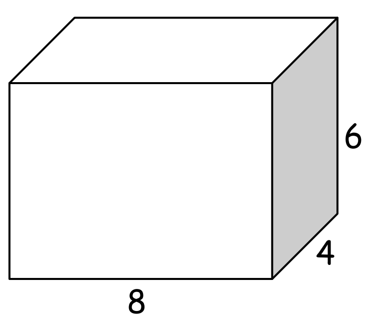
Lateral area: $Ph = 24(12) = 288\text{ cm}^2$.

To find the total surface area, $2B$ must be added.

Area of the (triangular)base: $\frac{1}{2}bh \rightarrow \frac{1}{2}(6)(8) \rightarrow 24$.

Total surface area: $288 + 2(24) = 336\text{ units}^2$.

b) Find the total surface area of the rectangular prism.

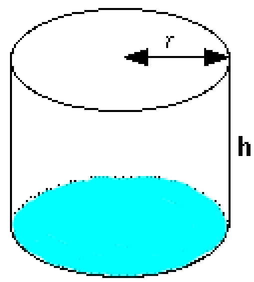


$$\begin{aligned} S &= 2lw + 2lh + 2wh \\ &= 2(8)(4) + 2(8)(6) + 2(4)(6) \\ &= 64 + 96 + 48 \\ &= 208\text{ units}^2. \end{aligned}$$

c) Find the width of a rectangular solid with length 15 cm, height 8 cm, and lateral area 400 cm^2.

$LA = Ph$ and $P = 2l + 2w$, so
$LA = (2l + 2w)h$.
$400 = [2(15) + 2w]8$
$50 = 30 + 2w$
$20 = 2w \rightarrow w = 10$. The width is 10 cm.

Cylinder – A solid with congruent circular bases that lie in parallel planes. The *altitude*, or *height*, of a cylinder is the perpendicular distance between its bases. The radius of the base is also called the *radius* of the cylinder.



Surface Area of a Right Cylinder Theorem: The surface area S of a right cylinder is $S = 2B + Ch = 2\pi r^2 + 2\pi rh$, where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height. $\{S = 2\pi r^2 + 2\pi rh$ and $LA = 2\pi rh\}$

Although it looks a little intimidating… this formula only requires 2 pieces of information, the *radius* and the *height*.

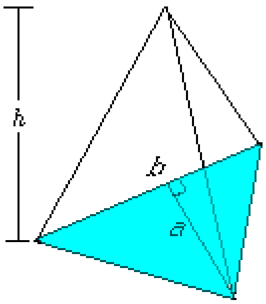
Example:

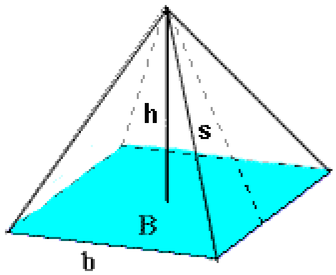
The total surface area of a cylinder is 256π cm^2. If $r = h$, find r and LA.

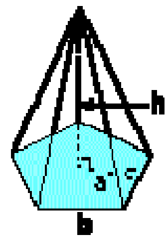
$256\pi = 2\pi r^2 + 2\pi rh$, since $r = h$
$256\pi = 2\pi r^2 + 2\pi r(r)$
$256\pi = 2\pi r^2 + 2\pi r^2$
$256\pi = 4\pi r^2$
$64 = r^2$
$r = 8$.

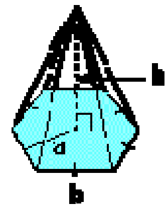
$LA = 2\pi rh$
$= 2\pi(8)(8)$
$= 128\pi$ units^2.

Pyramid – A polyhedron in which the base is a polygon and the *lateral faces* are triangles with a common *vertex*. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The *altitude*, or *height*, is the perpendicular distance between the base and the vertex.









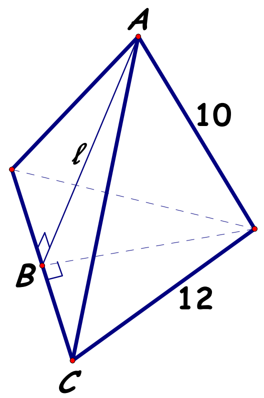
Slant height – the height of the lateral side of a pyramid or cone. Symbol: *l*.

It's nice to have a set of solids, preferably the plastic ones, to demonstrate the different parts of each one.
Plastic is nice because it can written on with overhead or whiteboard markers and erased afterwards.
It's also nice to have fillable ones to demonstrate the concept of three full pyramids fitting into a prism with the same base & height or three full cones fitting into a cylinder with the same base & height.
Beans or rice are better than sand or water!!!
Sand and water make a MESS!

Surface Area of a Regular Pyramid Theorem: The surface area *S* of a regular pyramid is $S = B + \frac{1}{2}Pl$, where *B* is the area of the base, *P* is the perimeter of the base, and *l* is the *slant height*. $\{S = B + \frac{1}{2}Pl$ and $LA = \frac{1}{2}Pl\}$

Again, **B** will be replaced with the area formula of the base shape.

Example: Find the lateral area and total surface area of the regular triangular pyramid.



To find the lateral area:

Perimeter of the base = 36.

In right ΔABC, $6^2 + l^2 = 10^2$, so $l = 8$.

$$LA = \frac{1}{2}Pl$$

$$= \frac{1}{2}(36)(8) = 144 \text{ units}^2$$

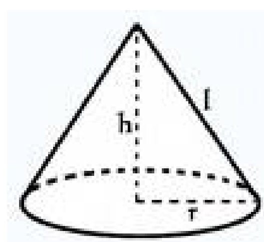
To find total surface area:

$S = B + \frac{1}{2}Pl$ where $B = \frac{1}{2}bh$ of an equilateral triangle and $\frac{1}{2}Pl$ is the LA. $B = \frac{1}{2}bh = \frac{1}{2}(12)\left(6\sqrt{3}\right) = 36\sqrt{3}$.

$S = 144 + 36\sqrt{3} \text{ units}^2$.

> Use 30°-60°90° triangle relationship to find the height of the base triangle in the previous example. Short leg is half of 12, or 6, so long leg is $6\sqrt{3}$.

Cone – A solid with a circular *base* and a *vertex* that is not in the same plane as the base. The *lateral surface* consists of all segments that connect the vertex with points on the edge of the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the plane that contains the base.



Surface Area of a Right Cone Theorem: The surface area S of a right cone is $S = \pi r^2 + \pi rl$, where r is the radius of the base and l is the *slant height*. $\{S = \pi r^2 + \pi rl$ and $LA = \pi rl\}$

Examples:

a) A cone has lateral area 100π and total surface area 136π. Find its radius.

Since $S = \pi r^2 + \pi rl$ and $LA = \pi rl$,

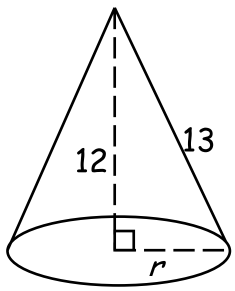
$$136\pi = \pi r^2 + 100\pi$$

$$36\pi = \pi r^2$$

$$36 = r^2$$

$$r = 6.$$

b) Find the lateral area and total surface area of the cone.



Use the Pythagorean Theorem (or a Pythagorean triple) to find r.

$h^2 + r^2 = l^2$

$12^2 + r^2 = 13^2$

$r = 5.$

$LA = \pi r l = \pi(5)(13) = 65\pi$

$S = \pi r^2 + \pi r l$ or $\pi r^2 + LA$

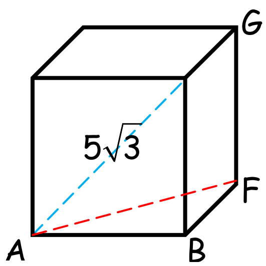
$= \pi(5)^2 + 65\pi = 90\pi$ units^2.

Make sure students are aware of the proper units on the answers to volume questions. Answers should always be cubic units.

Volume Postulates

Volume of a Cube Postulate: The volume of a cube is the cube of the length of its side, or $V = s^3$. $\{V = s^3\}$

Example: Find the volume of the cube.



a

AG is given as $5\sqrt{3}$ cm.

Since all sides of a cube are equal we will call them x. ΔABF is 45°-45°-90 triangle so $AF = x\sqrt{2}$. Using the Pythagorean Theorem on right ΔAFG results in:

$(5\sqrt{3})^2 = (x\sqrt{2})^2 + x^2$

$75 = 2x^2 + x^2$

$75 = 3x^2$

$25 = x^2$

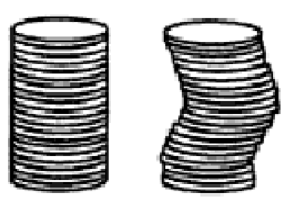
$x = 5.$

$V = s^3$

$= 5^3 = 125\,cm^3$.

Oblique Solid – A solid whose lateral edges are not perpendicular to its base(s).

Cavalieri's Principle Theorem: If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.



Suppose there are two neat stacks of pennies, 20 in each stack.
Then one stack is pushed to distort it as shown above.
How do the volumes of the two stacks compare? The volumes remain the same.

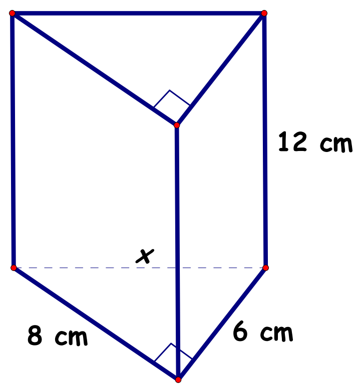
Volume of a Prism Theorem: The volume V of a prism is $V = Bh$, where B is the area if a base and h is the height. $\{V = Bh\}$

A method for finding the Volume of a Right Rectangular Prism:
$V = lwh$

Remind students that **B** will be replaced with the area formula of the base shape.

Examples:

a) Find the volume of the triangular prism.



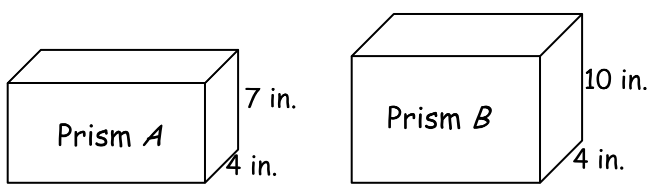
Right triangular base with area:

$A = \frac{1}{2}bh = \frac{1}{2}(6)(8) = 24.$

$V = Bh$

$= (24)(12) = 288\,\text{cm}^3.$

b) Prisms A and B have the same length and width, but different heights. If the volume of Prism B is 150 cubic inches greater than the volume of Prism A, what is the length of each prism?



Using the volume formula: $V = lwh$, Prism A has a volume of $28l$ and Prism B has a volume of $40l$. So $28l + 150 = 40l$, $150 = 12l$ and $l = 12.5$ inches.

Volume of a Cylinder Theorem: The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of the base, h is the height, and r is the radius of the base. $\{V = \pi r^2 h\}$

Examples:

a) The volume of a cylinder is 375π. If $h = 15$, find the lateral area.

$V = \pi r^2 h$

$375\pi = \pi r^2 (15)$

$25 = r^2$

$r = 5.$

$LA = 2\pi rh$

$= 2\pi(5)(15)$

$= 150\pi\,\text{units}^2.$

b) The lateral area of a cylinder is 96π. If $r = 8$, find the volume.

$LA = 2\pi rh$

$96\pi = 2\pi(8)h$

$96\pi = 16\pi h$

$h = 6.$

$V = \pi r^2 h$

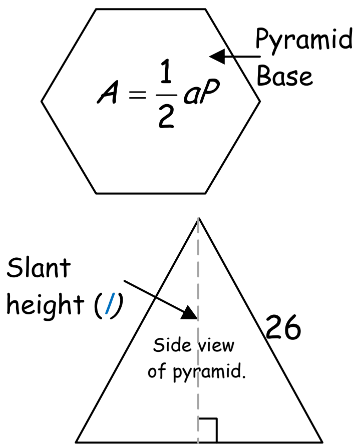
$= \pi(8)^2(6)$

$= 384\pi \text{ units}^3.$

Volume of a Pyramid Theorem: The volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. $\{V = \frac{1}{3}Bh\}$

Example:

Find the volume of a regular hexagonal pyramid with base edge 20 and lateral edge 26.



Perimeter of base = $6(20) = 120$.

To find the apothem length, the 30°-60°-90° relationship must be used . The short leg is one half the side of the hexagon, or 10. The apothem is the long leg so its length is $10\sqrt{3}$. Area of base is $A = \frac{1}{2}(10\sqrt{3})(120) = 600\sqrt{3}$.

Use the Pythagorean Theorem to find the slant height $l^2 + 10^2 = 26^2$ (or a multiple of the 5-12-13 triple), $l = 24$.

$V = \frac{1}{3}Bh = \frac{1}{3}(600\sqrt{3})(24) = 4800\sqrt{3} \text{ units}^3.$

Volume of a Cone Theorem: The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$, where B is the area if the base, h is the height, and r is the radius of the base. $\{V = \frac{1}{3}\pi r^2 h\}$

Examples:

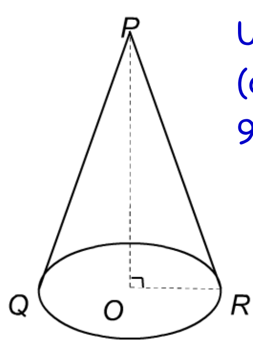
a) A cone has volume $432\pi \text{ cm}^3$ and height 9 cm. Find its slant height (PR).

$V = \frac{1}{3}\pi r^2 h$

$432\pi = \frac{1}{3}\pi r^2(9)$

$144 = r^2$

$r = 12.$



Use the Pythagorean Theorem to find l.

(or a multiple of the 3-4-5 right triangle)

$9^2 + 12^2 = l^2 \rightarrow l = 15 \text{ cm} \rightarrow PR = 15 \text{ cm}.$

b) A cone and a cylinder both have height 7 and radius 3. Find the ratio of their volumes without actually calculating them.

The ratio of the volume of the cylinder to the cone (with the same height and radius) is $3:1$. This is based solely on the formulas, it takes three cones to fill a cylinder of same height and radius. Students could check their work by calculating the volumes: $V(cylinder) = \pi r^2 h = \pi(3)^2(7) = 63\pi$ and $V(cone) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(7) = 21\pi$. The corresponding ratio would be $\frac{63\pi}{21\pi} = \frac{3}{1}$.

Sphere – The set of all points in space that are a given distance from a point, called the *center* of the sphere.

Great Circle – the intersection of a sphere and a plane that contains the center of the sphere. It is the largest circle that can be drawn on any sphere.

Spheres are awesome!!! The only measurement needed is the radius!

But it does have cubes, and cube roots. Here are the cubes of some common numbers:
$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$ $6^3 = 216$ $7^3 = 343$.
Also, review how to take the cube root on the calculator for unfamiliar values.
Most often, it's the number ^ (1/3). Have them try it out on their calculators.

Surface Area of a Sphere Theorem: The surface area S of a sphere with radius r is $S = 4\pi r^2$. $\{S = 4\pi r^2\}$

Example:

The surface area of a sphere is $\frac{\pi}{4}$. Find its diameter.

$$S = 4\pi r^2$$
$$\frac{\pi}{4} = 4\pi r^2$$
$$\frac{1}{16} = r^2$$
$$r = \frac{1}{4}. \qquad d = \frac{1}{2}\text{units.}$$

Volume of a Sphere Theorem: The volume V of a sphere with radius r is $V=\frac{4}{3}\pi r^3$.

$\{V=\frac{4}{3}\pi r^3\}$

Examples:

a) The volume of a sphere is 972π. Find its surface area.

$$V=\frac{4}{3}\pi r^3$$
$$972\pi=\frac{4}{3}\pi r^3$$
$$729=r^3$$
$$r=9.$$

$$S=4\pi r^2$$
$$=4\pi(9)^2$$
$$=324\pi\text{ units}^2.$$

b) A hemisphere ("half" a sphere) has a radius 6. A sphere has a radius of 3. Compare their volumes.

Hemisphere

$$V=\left(\frac{4}{3}\pi r^3\right)\frac{1}{2}$$
$$=\frac{4}{3}\pi(6)^3\left(\frac{1}{2}\right)$$
$$=144\pi\text{ units}^3.$$

Sphere

$$V=\frac{4}{3}\pi r^3$$
$$=\frac{4}{3}\pi(3)^3$$
$$=36\pi\text{ units}^3.$$

The hemisphere has a volume 4 times larger than the sphere.

Here's the math, the $\frac{4}{3}\pi$ is the same for both, the difference is…

$\frac{1}{2}(6)^3=\frac{1}{2}(2\cdot 3)^3=\frac{1}{2}\cdot 2\cdot 3\cdot 2\cdot 3\cdot 2\cdot 3=3\cdot 2\cdot 3\cdot 2\cdot 3=108$ and

$(3^3)=3\cdot 3\cdot 3=27$ and… $27(4)=108$. TAH DAH!!!

Formulas for the lateral area *LA*, total surface area *S* and volume *V* of the more common solids are given below.

PRISM	CYLINDER	PYRAMID	CONE	SPHERE
perimeter of base *P*, height *h*, Area of Base *B*, length *l* and width *w*	radius *r* and height *h*	perimeter of base *P*, slant height *l*, Area of Base *B*, and height *h*	radius *r*, slant height *l*, and height *h*	radius *r*
$LA = Ph$	$LA = 2\pi rh$	$LA = \frac{1}{2}Pl$	$LA = \pi rl$	
$S = 2B + Ph$ $S = LA + 2B$ *Rectangular Prism:* $S = 2lw + 2lh + 2wh$ Cube: $S = 6s^2$	$S = 2\pi r^2 + 2\pi rh$	$S = B + \frac{1}{2}Pl$	$S = \pi r^2 + \pi rl$	$S = 4\pi r^2$
$V = Bh$ *Rectangular Prism:* $V = lwh$ Cube: $V = s^3$	$V = \pi r^2 h$	$V = \frac{1}{3}Bh$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$

In a PRISM or PYRAMID: Area of Base *(B)*… will be replaced with the appropriate area formula, to match the shape of the base.	
SQUARE	$A = s^2$
RECTANGLE	$A = bh$
TRIANGLE	$A = \frac{1}{2}bh$
PARALLELOGRAM	$A = bh$
TRAPEZOID	$A = \frac{1}{2}h(b_1 + b_2)$
RHOMBUS/KITE	$A = \frac{1}{2}d_1 d_2$
EQUILATERAL TRIANGLE	$A = \frac{1}{4}s^2\sqrt{3}$
REGULAR POLYGON	$A = \frac{1}{2}aP$

Syllabus Objective: 9.3 - The student will solve real world problems of surface area and volume.

Examples:

a) A cylindrical can of baked potato chips has a height of 27 centimeters and a radius of 4 centimeters. A new can is advertised as being 30% larger than the regular can. If both cans have the same radius, what is the height of the larger can?

If the can is 30% larger, its volume is 30% greater. The volume the smaller can would be $V = \pi r^2 h = \pi(4)^2(27) = 432\pi \approx 1357.2\,\text{cm}^3$.

An increase of 30% would result in the larger can having a volume of $561.6\pi \approx 1764.3\,\text{cm}^3$. Therefore:

$$V = \pi r^2 h$$
$$561.6\pi = \pi(4)^2 h$$
$$h = 35.1\,\text{cm}.$$

b) The dimensions of two conical canvas tepees are shown in the table. Not including the floors, approximately how much more canvas is used to make Tepee B than Tepee A?

Tepee	**Diameter (ft)**	**Height (ft)**
A	14	6
B	20	9

Tepee A **Tepee B**

Use the Pythagorean Theorem to find the slant height:

Tepee A:

$$6^2 + 7^2 = l^2$$
$$l = \sqrt{85}$$
$$LA = \pi r l$$
$$= \pi(7)(\sqrt{85})$$
$$\approx 202.75\,\text{ft}^2.$$

Tepee B:

$$9^2 + 10^2 = l^2$$
$$l = \sqrt{181}$$
$$LA = \pi r l$$
$$= \pi(10)(\sqrt{181})$$
$$\approx 422.66\,\text{ft}^2.$$

$$422.66 - 202.75 = 219.91\ \text{ft}^2.$$

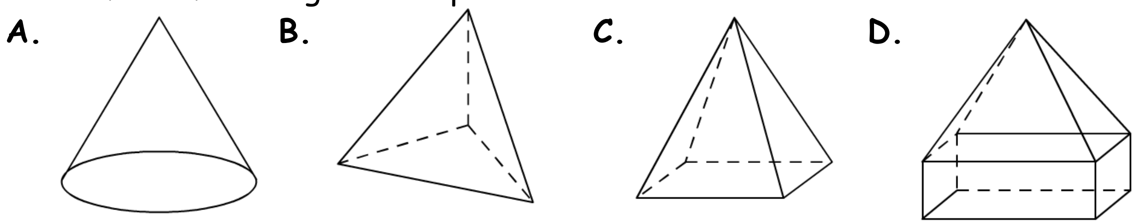
Tepee B uses 219.91 ft^2 more canvas.

Volume Congruence Postulate: If two polyhedra are congruent, then they have the same volume.

Volume Addition Postulate: The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

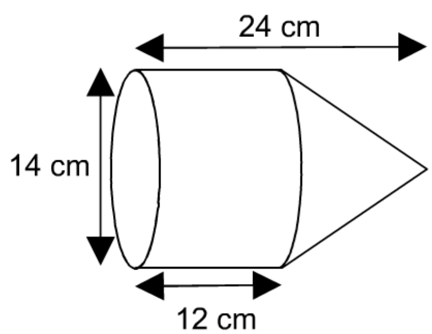
Examples:

a) Which of the following is a composite solid?



Solution: D

b) The diagram shows a solid consisting of a cylinder and a cone. Calculate the volume in cm^3, of the solid.



The plan would be: calculate the volume of the cylinder, $V = \pi r^2 h$, add the volume of the cone, $V = \frac{1}{3}\pi r^2 h$. The height of the cylinder and the cone are both 12 cm, the radius of each is 7 cm.

$$V = \pi r^2 h + \frac{1}{3}\pi r^2 h$$
$$= \pi(7)^2(12) + \frac{1}{3}\pi(7)^2(12)$$
$$= 784\pi \text{ units}^3.$$

Syllabus Objective: 9.4 - The student will solve area and volume problems of similar two and three dimensional figures.

Similar Solids Theorem: If two similar solids have a scale factor of $a:b$, then corresponding areas have a ratio of $a^2:b^2$, and the corresponding volumes have a ratio of $a^3:b^3$.

Examples:

a) Are the given solids similar?

i. Two regular square pyramids have heights 10 and 12. The bases are squares with sides 4 and 4.8 respectively.

All squares are similar and $\frac{10}{12} = \frac{4}{4.8} \rightarrow YES.$

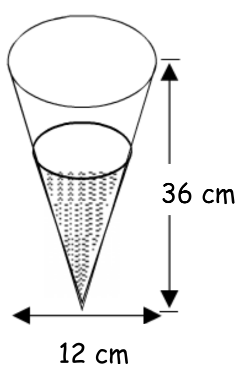
ii. One rectangular solid has a length 7, width 5, and height 3. Another rectangular solid has length 14, width 10, and height 9.

$\frac{7}{14} = \frac{5}{10} \neq \frac{3}{9} \rightarrow NO.$

iii. Two right triangular prisms have heights 3 and 6. Their bases are triangles with sides 3, 4, 5, and 6, 8, 10, respectively.

$\frac{3}{6} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \rightarrow YES.$

b) Find the volume of the shaded portion of the cone. The larger cone is similar to the smaller cone and twice as tall.



Since the cones are similar, and the heights have a ratio of $2:1$, the volumes will have a ratio of $8:1$.

The plan would be to find the volume of the larger one and solve the resulting proportion.

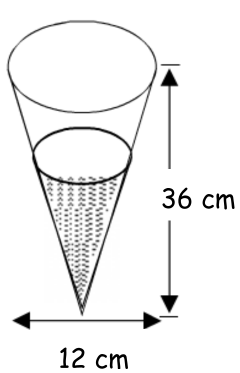
$V = \frac{1}{3}\pi r^2 h \rightarrow \frac{1}{3}\pi(6)^2(36) = 432\pi.$

$\frac{8}{1} = \frac{432\pi}{x}$, where x is the volume of the shaded cone.

$8x = 432\pi$

$x = 54\pi.$ $\qquad$ $V = 54\pi \text{ cm}^3.$

c) Find the volume of the unshaded portion of the cone. The larger cone is similar to the smaller cone and twice as tall.



Since the cones are similar, their measurements are in proportion. If the height of the larger one is twice the height of the smaller one, then the diameter of the larger one is twice the diameter of the smaller one.

The plan would be to find the volume of the larger cone and subtract the volume of the smaller cone.

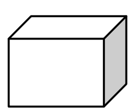
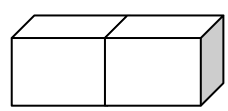
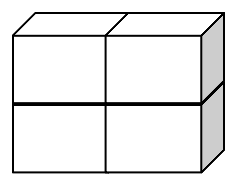
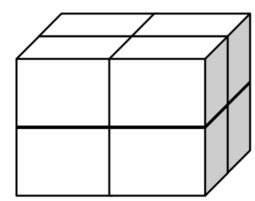
$$V = \frac{1}{3}\pi r^2 h \rightarrow \frac{1}{3}\pi(6)^2(36) - \frac{1}{3}\pi(3)^2(18) = 378\pi \text{ units}^3.$$

Obviously, if example (a) has already been worked out, students need only subtract $432\pi - 54\pi$.

d) Complete the table, which refers to two similar prisms.

Scale Factor	Ratio of Base Perimeters	Ratio of Heights	Ratio of Lateral Areas	Ratio of Surface Areas	Ratio of Volumes
2 : 5	??? 2 : 5	??? 2 : 5	??? 4 : 25	??? 4 : 25	??? 8 : 125
??? 1 : 3	??? 1 : 3	1 : 3	??? 1 : 9	??? 1 : 9	??? 1 : 27
??? 2 : 7	??? 2 : 7	??? 2 : 7	4 : 49	??? 4 : 49	??? 8 : 343
??? 5 : 6	??? 5 : 6	??? 5 : 6	??? 25 : 36	??? 25 : 36	125 : 216
??? 3 : 10	??? 3 : 10	??? 3 : 10	??? 9 : 100	??? 9 : 100	27 : 1000

Model the effect of doubling dimensions with stacked blocks, text books or small boxes; show students how, as measurements double or triple, lateral area, surface area and volume will change.
*Students can visualize the overlapping areas and growing capacity.

				
Dimensions:	*l, w, h*	length doubled	& height doubled	& width doubled
		Not Similar	Not Similar	SIMILAR
Lateral Area:	*LA*	→	→	4*LA* → quadruple
Surface Area:	*S*	→	→	4*S* → quadruple
Volume:	*V*	→	→	8*V* → octuple

****Sculpt Big or Sculpt Small?****

The Problem:
A sculptor has the molds to cast similar solid bronze figures of height either 20 cm or 40 cm. She can sell these figures for $30 and $150 respectively. If her main cost is the cost of the bronze, which size figure is it more profitable for her to make? What difference might it make to your advice if the figures were hollow?

Solutions:

A

This problem uses reasoning based on Area and Volume scale factors.

Linear scale factor of figures $= 40 : 20 = 2 : 1.$

So Volume scale factor $= 2^3 : 1^3 = 8 : 1.$

Price "scale factor" for the same figures $= 150 : 30 = 5 : 1.$

Therefore, if she makes and sells the larger figures, the sculptor gets only 5 times the money for 8 times the volume of bronze.

She would be advised to stick to making and selling the smaller figures only.

B

If the figures were largely hollow, the quantities of bronze required in their manufacture would be more closely modeled by their *surface areas* than their *volumes*, and therefore:

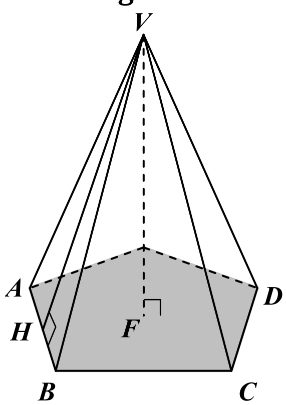
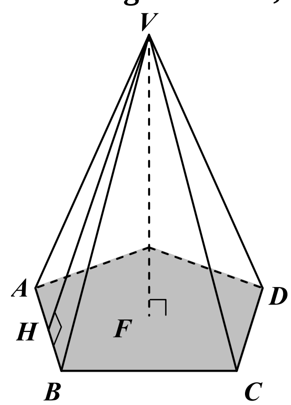
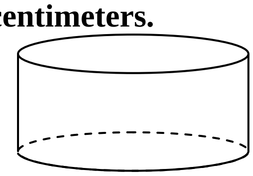
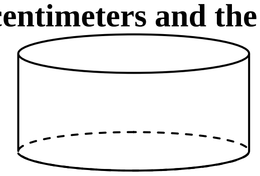
The relevant Surface Area scale factor $= 2^2 : 1^2 = 4 : 1.$

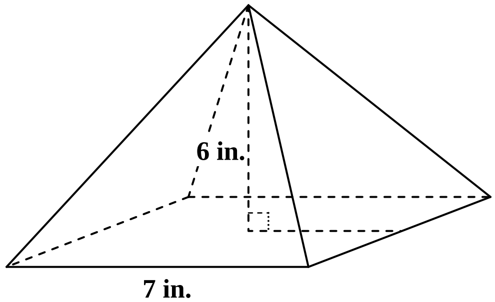
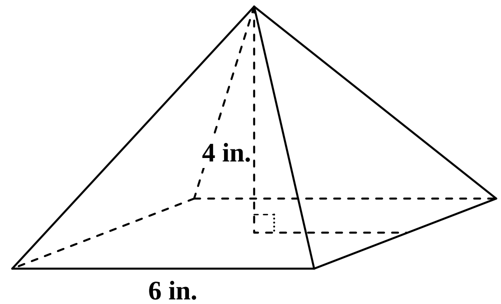
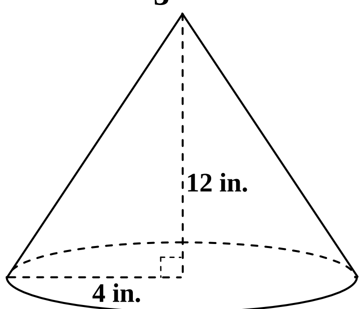
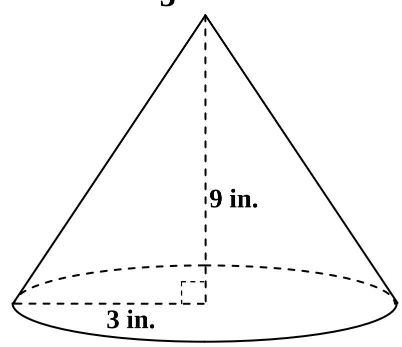
Therefore, if she makes and sells the larger figures, the sculptor gets 5 times the money for only 4 times the amount of bronze.

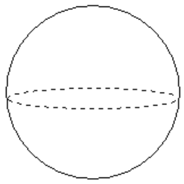
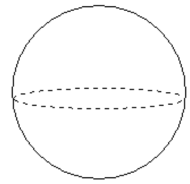
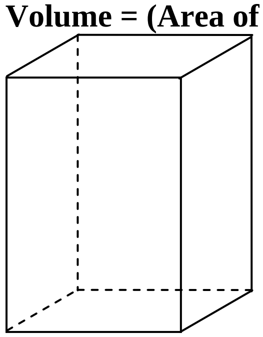
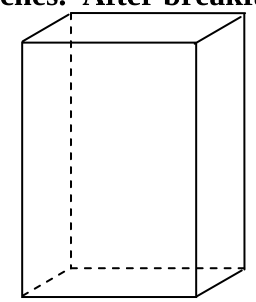
If this is the case, she would be advised to make the larger figures only.

This unit is designed to follow the Nevada State Standards for Geometry, CCSD syllabus and benchmark calendar. It loosely correlates to Chapter 12 of McDougal Littell *Geometry* © 2004, sections 12.1 – 12.7. The following questions were taken from the 2nd semester common assessment practice and operational exams for 2008-2009 and would apply to this unit.

Multiple Choice

#	Practice Exam (08-09)	Operational Exam (08-09)
5.	**Given the figure below:**  **What is the best description of $\overline{VF}$?** **A.** altitude **B.** base edge **C.** lateral edge **D.** slant height	**Given the figure below,**  **What is the best description of $\overline{VH}$ in relation to the pyramid?** **A.** base edge **B.** height **C.** lateral edge **D.** slant height
6.	**The surface area of a cylinder is 2 × (Area of Base) + (Circumference of the Base) × height.** **In the cylinder below, the radius is 4 centimeters and surface area is 72π square centimeters.**  **What is the height of the cylinder?** **A.** 4 cm **B.** 5 cm **C.** 6 cm **D.** 9 cm	**The surface area of a cylinder is:** **2 × (Area of Base) + (Circumference of the Base) × height** **In the cylinder below, the radius is 5 centimeters and the height is 3 centimeters.**  **What is the surface area of the cylinder in terms of π?** **A.** 30π cm^2 **B.** 50π cm^2 **C.** 75π cm^2 **D.** 80π cm^2

7.	**A regular pyramid has height of 6 inches and the measure of the base edge is 7 inches.** **Volume =** $\frac{1}{3}$ **× (Area of Base) × height**  **What is the volume of the pyramid?** **A.** 49 $in.^3$ **B.** 98 $in.^3$ **C.** 147 $in.^3$ **D.** 294 $in.^3$	**A square pyramid has height of 4 inches and the measure of the base edge is 6 inches.** **Volume =** $\frac{1}{3}$ **× (Area of Base) × height**  **What is the volume of the pyramid?** **A.** 16 $in.^3$ **B.** 48 $in.^3$ **C.** 96 $in.^3$ **D.** 144 $in.^3$
8.	**What is the volume of the cone below?** **Volume =** $\frac{1}{3}$ **× (Area of Base) × height**  **A.** 192π $in.^3$ **B.** 96π $in.^3$ **C.** 64π $in.^3$ **D.** 48π $in.^3$	**A cone has a height of 9 inches and a radius of 3 inches.** **Volume =** $\frac{1}{3}$ **× (Area of Base) × height**  **What is the volume of the cone in terms of** π**?** **A.** 108π $in.^3$ **B.** 81π $in.^3$ **C.** 27π $in.^3$ **D.** 18π $in.^3$

9.	**A group of students wants to make a fabric toy ball to donate to the canine rescue. The diameter of the ball is 3 inches.** **Surface area = 4 × (Area of a Great Circle).**  **Approximately how many square inches of fabric will they need for each ball?** **A.** 29 $in.^2$ **B.** 57 $in.^2$ **C.** 76 $in.^2$ **D.** 114 $in.^2$	**The diameter of a softball is approximately four inches.** **Surface area of a Sphere = 4 × (Area of a Great Circle)**  **Approximately how many square inches of leather are needed to cover the ball?** **A.** 15 $in.^2$ **B.** 50 $in.^2$ **C.** 85 $in.^2$ **D.** 250 $in.^2$
10.	**A cereal box is 18 inches by 3 inches by 12 inches. After breakfast, the box is one-third full.** **Volume = (Area of Base) × height**  **How many cubic inches of cereal are left inside?** **A.** 36 $in.^3$ **B.** 72 $in.^3$ **C.** 216 $in.^3$ **D.** 648 $in.^3$	**A cereal box is 10 inches by 2 inches by 15 inches. After breakfast, the box is half full.**  **How many cubic inches of cereal are left inside?** **A.** 75 $in.^3$ **B.** 150 $in.^3$ **C.** 300 $in.^3$ **D.** 600 $in.^3$