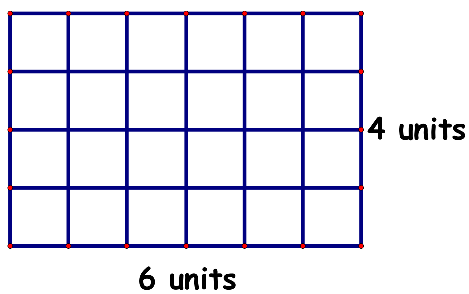
Geometry Unit 8 - Notes
Perimeter and Area

Syllabus Objective: 8.1 - The student will formulate strategies for finding the perimeter or area of various geometric figures.

Review terms: *1) area* *2) perimeter* *3) regular polygons*

Area of a Rectangle Theorem: The area of a rectangle is the product of its base and its height. $\{A = bh\}$

Example: Find the perimeter and area of the rectangle.



4 rows by 6 columns

This rectangle has a base length of 6 and a height (width) of 4.

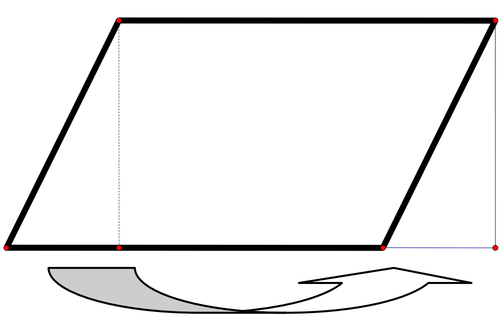
$P = 2l + 2w = 2(6) + 2(4) = 12 + 8 = 20$ units.

$A = bh = (6)(4) = 24$ units^2.

Make sure students are aware of the proper units on the answers to area questions. Answers should always be square units.

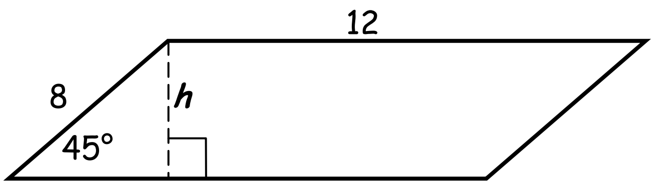
Area of a Square Postulate: The area of a square is the square of the length of its side, or $A = s^2$.

Area of a Parallelogram Theorem: The area of a parallelogram is the product of a base and its corresponding height. $\{A = bh\}$



Notice the triangle formed when the height is drawn on the left end of the parallelogram. If you were to cut that off and affix it to the right end of the parallelogram, you would form a *rectangle*. The rectangle would have the same base length and the same height as your parallelogram, and their areas would be equal! As you can see, those formulas are the same.

Example: Find the perimeter and area of the parallelogram.



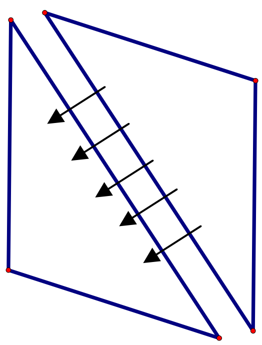
$P = 2l + 2w = 2(12) + 2(8) = 24 + 16$
$= 40$ units.

Since h is a leg of a $45\circ 45\circ 90$ triangle:

$$h = \frac{\text{hypotenuse}}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$$A = bh = (12)(4\sqrt{2}) = 48\sqrt{2} \text{ units}^2.$$

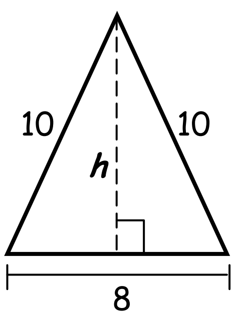
Area of a Triangle Theorem: The area of a triangle is one half the product of a base and its corresponding height. $\{A = \frac{1}{2}bh\}$



Any triangle can be copied and rotated 180°. When the original and the copy are joined together, a *parallelogram* is formed. Since both triangles have the same area, the area of that parallelogram is twice as large as the original triangle. The parallelogram formula can be used, but it must be divided in half to find the area of the triangle.

$$A = \frac{1}{2}bh$$

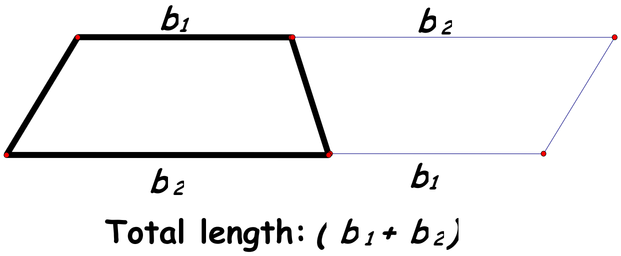
Example: Find the perimeter and area of the triangle.



The perimeter is found by $P = a + b + c = 10 + 10 + 8 = 28$ units. The altitude to the base of an isosceles triangle bisects the base. Use the Pythagorean Theorem to find h.

$$4^2 + h^2 = 10^2$$
$$h^2 = 100 - 16 = 84$$
$$h = 2\sqrt{21}$$
$$A = \frac{1}{2}bh = \frac{1}{2}(8)(2\sqrt{21}) = 8\sqrt{21} \text{ units}^2.$$

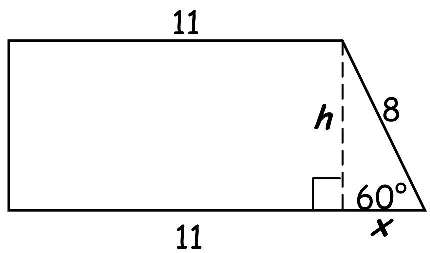
Area of a Trapezoid Theorem: The area of a trapezoid is one half the product of the height and the sum of the bases. $\{A = \frac{1}{2}h(b_1 + b_2)\}$



Notice what happens when you copy the trapezoid, rotate it 180° and affix it to the end of the trapezoid. A *parallelogram* is formed! It has the same height as the trapezoid, but it will have an area twice as big. Also notice that the base of the parallelogram is the total length $\underline{b_1 + b_2}$.

$$A = \frac{1}{2}h(b_1 + b_2)$$

<u>Example:</u> **<u>Find the perimeter and area of the trapezoid.</u>**



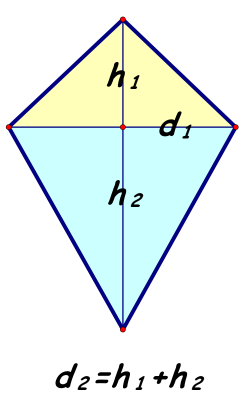
x is the shorter leg of a $30^\circ 60^\circ 90$ triangle, so $x = \frac{1}{2}(8) = 4$. The longer base of the trapezoid is $x + 11$ or 15. The height is the longer leg of a 30°-60°-90° triangle, so $h = 4\sqrt{3}$.

$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(4\sqrt{3})(15 + 11) \\ &= 52\sqrt{3}\text{ units}^2. \end{aligned}$$

The perimeter is found by adding the four sides, using the measure of h for the unknown side.

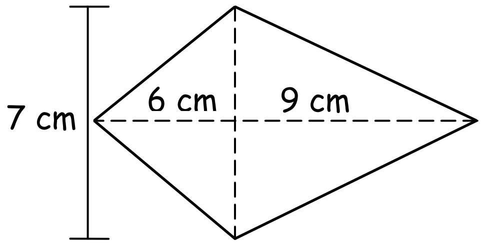
$P = 11 + 8 + 15 + 4\sqrt{3} = 34 + 4\sqrt{3}$ units.

Area of a Kite Theorem: The area of a kite is one half the product of the lengths of its diagonals. $\{A = \frac{1}{2}d_1 d_2\}$



In order to find the area of a kite, we split it into *2 triangles*. The top triangle has an area of $\frac{1}{2}d_1 h_1$ and the bottom triangle has an area of $\frac{1}{2}d_1 h_2$. By adding these two areas together we obtain the area of the entire kite, $\frac{1}{2}d_1 h_1 + \frac{1}{2}d_1 h_2 \to \frac{1}{2}d_1(h_1 + h_2) \to \frac{1}{2}d_1 d_2$.

Example: Find the perimeter and area of the kite.



The area is found by the formula $A=\frac{1}{2}d_1d_2$. $A=\frac{1}{2}(7)(15)=52.5\text{units}^2$.

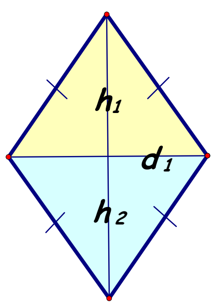
To find the perimeter the Pythagorean Theorem must be used.

$$\text{each of 2 shorter sides}=\sqrt{6^2+3.5^2}=\sqrt{48.25}=\sqrt{\frac{193}{4}}=\frac{\sqrt{193}}{2}$$

$$\text{each of 2 longer sides}=\sqrt{9^2+3.5^2}=\sqrt{93.25}=\sqrt{\frac{373}{4}}=\frac{\sqrt{373}}{2}$$

$$P=2(\text{shorter sides})+2(\text{longer sides})=2\left(\frac{\sqrt{193}}{2}\right)+2\left(\frac{\sqrt{373}}{2}\right)=\sqrt{193}+\sqrt{373}\text{ units.}$$

Area of a Rhombus Theorem: The area of a rhombus is equal to one half the product of the lengths of the diagonals. $\{A=\frac{1}{2}d_1d_2\}$

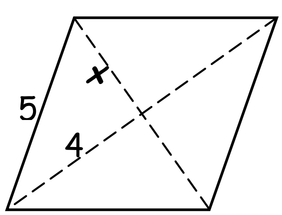


$d_2=h_1+h_2$

The same reasoning used to show the derivation of the kite formula can be used to show the derivation of the rhombus formula. The 2 triangles are congruent as well.

I like to take this opportunity to review the quadrilateral properties; it allows students to interchange formulas when different information is given. For example, the rhombus formula can be used to find the area of a square because every square is a rhombus.

Example: Find the perimeter and area of the rhombus.



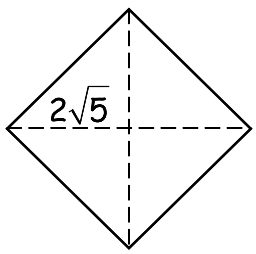
Perimeter $=4s$, because all sides are congruent. $P=4(5)=20\text{units}$.

Remember the diagonals of a rhombus are $\perp$ bisectors of each other.

$5^2=4^2+x^2$, so $x=3$.

$$A=\frac{1}{2}d_1d_2$$
$$=\frac{1}{2}(2\cdot4)(2\cdot3)$$
$$=24\text{units}^2.$$

Example: **Find the perimeter and area of the square.**



Given the length of a diagonal or piece of a diagonal, it is easier for students to use the rhombus formula to find the area than the square formula that requires a side length. Both diagonals of this square would be $4\sqrt{5}$ in length.

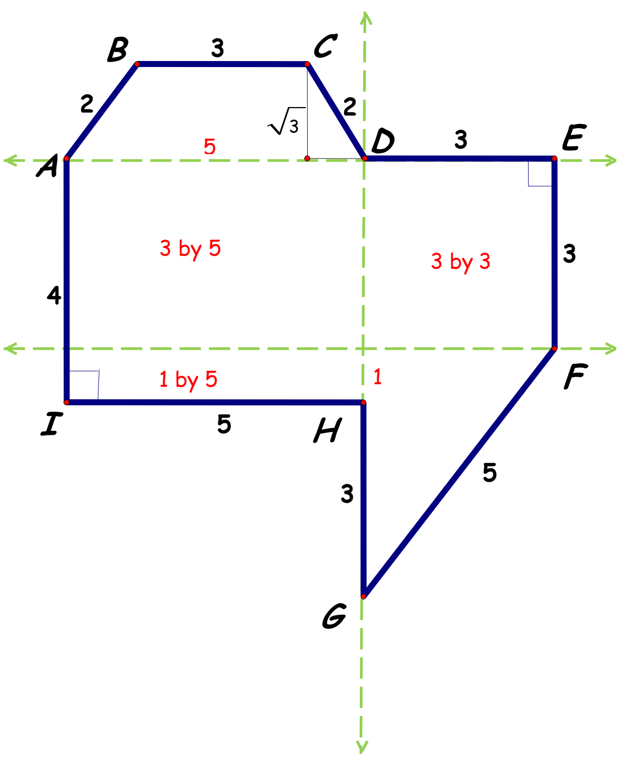
$$A = \frac{1}{2}d_1d_2 = \frac{1}{2}\left(4\sqrt{5}\right)\left(4\sqrt{5}\right) = \frac{1}{2}(16)(5) = 40\text{units}^2.$$

If the area is 40 units^2, then each side is $\sqrt{40} = 2\sqrt{10}$. And the perimeter would be $P = 8\sqrt{10}$ units.

Area Addition Postulate: The area of a region is the sum of the areas of its nonoverlapping parts.

Have students be creative in sectioning unusual figures so that they can easily determine the perimeter and area of individual pieces. Students can then add the areas together, but they need to be careful to not double count pieces when calculating perimeters or areas.

Example: **Find the perimeter and area of the figure below:**



Solution:
Students can create a trapezoid, two rectangles, a square, and a triangle (as suggested by the dotted lines). Find the area of each and then add together.

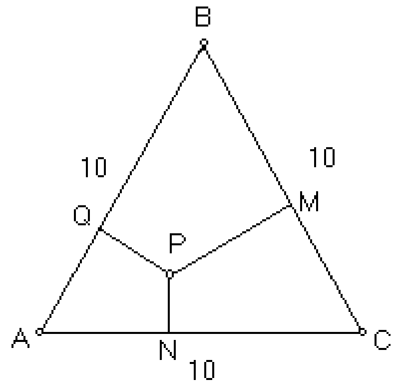
Perimeter $= 30$ units.

Area $= 35 + 4\sqrt{3}$ units^2.

P where you like

The Problem:

Draw an equilateral triangle, *ABC*, of side 10cm.
Choose any point in the interior of the triangle, and call it *P*.
Drop perpendiculars from *P* to each of the three sides.
These points of contact can be called *M*, *N* and *Q*.
Measure the lengths of *PM*, *PN* and *PQ*.
What is the **total** of these three lengths
PM + *PN* + *PQ*?
Try it again with some other point *P'*.



Can you explain why the total is always the same?

Hint:
Draw some inner triangles, and think about area…

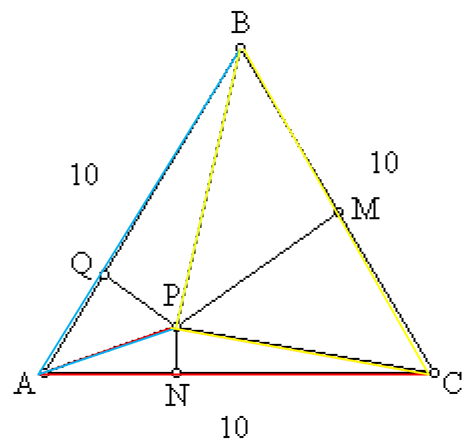
Solution:

The area of ΔABC is $\frac{1}{2}(10)(height)$, height $= 5\sqrt{3}$.

The area of ΔACP is $\frac{1}{2}(10)(PN)$,

ΔCBP is $\frac{1}{2}(10)(PM)$,

ΔBAP is $\frac{1}{2}(10)(PQ)$.



The total area of the three triangles $= \frac{1}{2}(10)(PN + PM + PQ)$..

And $\frac{1}{2}(10)(PN + PM + PQ) = \frac{1}{2}(10)(\text{height of original } \Delta)$..

So the value *PN* + *PM* + *PQ* must equal the height of the original triangle, $5\sqrt{3} \approx$ 8.66 cm.

Even when *P* is placed at a vertex itself, *PN* + *PM* + *PQ* ≈ 8.66 + 0 + 0…

Formulas for the perimeter *P*, area *A* and circumference *C* of some common plane figures are given below.

SQUARE	RECTANGLE	TRIANGLE	CIRCLE	PARALLELOGRAM	TRAPEZOID	RHOMBUS
side length s	base (or length) b and height (or width) h	side lengths a, b, and c, base b, and height h	radius r	base b and height h	height h, bases b_1 and b_2	diagonals d_1 and d_2
$P = 4s$	$P = 2b + 2h$ or $P = 2(b+h)$	$P = a + b + c$	$C = 2\pi r$			
$A = s^2$	$A = bh$	$A = \frac{1}{2}bh$	$A = 2\pi r^2$	$A = bh$	$A = \frac{1}{2}h(b_1 + b_2)$	$A = \frac{1}{2}d_1 d_2$

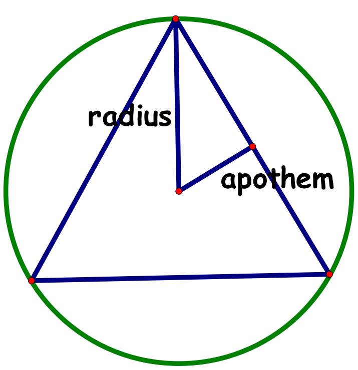
Area of an Equilateral Triangle Theorem: The area of an equilateral triangle is one fourth the square of the length of the side times $\sqrt{3}$. $\{A = \frac{1}{4}s^2\sqrt{3}\}$

<u>Example:</u> **<u>Find the area of an equilateral triangle.</u>**

With sides $6\sqrt{3}$: $A = \frac{1}{4}s^2\sqrt{3} = \frac{1}{4}(6\sqrt{3})^2\sqrt{3} = \frac{1}{4}(36)(3)\sqrt{3} = 27\sqrt{3}$ units^2.

<u>Apothem</u> of a regular polygon – the distance from the center to the midpoint of any side of the polygon.

<u>Radius</u> of a regular polygon – the distance from the center to any vertex of the polygon.



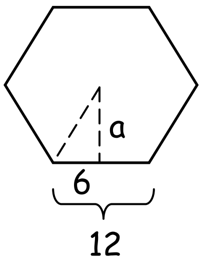
The radius is always longer than the apothem.

Area of a Regular Polygon Theorem: The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$ or $A = \frac{1}{2}ans$, where n is the number of sides. $\{A = \frac{1}{2}aP\}$

When dealing with equilateral triangles, squares, regular hexagons; trig ratios are not required.

Examples: **Find the area of the regular polygons.**

a) Regular hexagon with side 12.

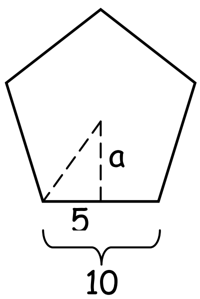


The central angle of a regular hexagon is 60°. The apothem will bisect that angle and create a 30°-60°-90° triangle. The shorter leg is 6 units long so the apothem is $6\sqrt{3}$.

$A=\frac{1}{2}aP$ or $\frac{1}{2}ans=\frac{1}{2}6\sqrt{3}(6)(12)=216\sqrt{3}\text{ units}^2$.

The perimeter is 6(12) or P = 72 units.

b) Regular pentagon with side 10. {will need trig ratio}



The central angle of a regular pentagon is 72°. The apothem will bisect that angle and side. To find the length of the apothem the tangent ratio must be used.

$\tan 36^\circ=\frac{5}{a}\rightarrow a=\frac{5}{\tan 36^\circ}\rightarrow A=\frac{1}{2}aP=\frac{1}{2}\left(\frac{5}{\tan 36^\circ}\right)(5\cdot 10)$,

$A\approx 172.05\text{ units}^2$.

The perimeter is 5(10) or P = 50 units.

Syllabus Objective: 8.2 - The student will solve problems using perimeter or areas of geometric figures.

Area Congruence Postulate: If two polygons are congruent, then they have the same area.

Areas of Similar Polygons Theorem: If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

> If students have trouble determining which ratio is given, have them think about the units that would be used to label the measurements given. If it would be a unit label, then it is an $a:b$ ratio, if it would be a unit^2 label, then it is an $a^2:b^2$ ratio.

Examples:

a) The lengths of two similar rectangles are 3 m and 7 m, respectively. What is the ratio of their areas?

The ratio of the sides is $a:b$ or $3:7$, the ratio of the areas is $a^2:b^2$ or $9:49$.

b) The areas of two circles are 25π and 100π.

The ratio of the areas is $a^2:b^2$ or $25\pi:100\pi$ reduced to $1:4$, therefore the ratio of the radii is $a:b$ or $5:10$ reduced to $1:2$.

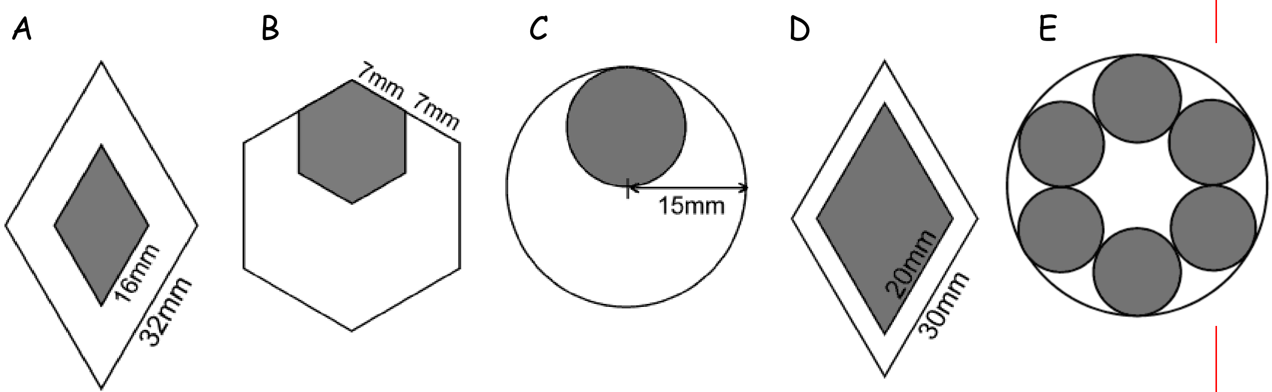
i. What is the ratio of their diameters? $10:20$ reduced to $1:2$.

ii. What is the ratio of their circumferences? $10\pi:20\pi$ reduced to $1:2$.

Pendants

The Problem:

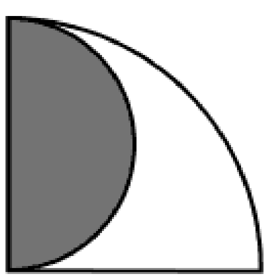
Shown below are several pendants, for use on necklaces. The plain areas are to be made from silver, and the shaded areas from gold.
Your task is to calculate the fraction, or proportion, of each pendant that is made from gold.



Solutions:

The first three pendants all have a gold area of $\frac{1}{2}$ the scale, and therefore $\frac{1}{4}$ the area, of the entire piece.
Pendant D's gold area is $\frac{2}{3}$ the scale, and therefore $\frac{4}{9}$ the area of the original piece.
Pendant E's smaller circles are $\frac{1}{3}$ the scale, and therefore $\frac{1}{9}$ the area of the original piece. Therefore the final gold proportion is $\frac{6}{9}$ or $\frac{2}{3}$.

The "nice" method that this problem draws attention to is the "scale factor squared" ratio for area enlargements.
Looking at the top right quarter of Pendant C gives a nice surprise.



Since the shaded semi-circle comprises two *quarters* of the entire quadrant (by the above reasoning), the shaded and white parts of the design are equal.

Circumference of a Circle Theorem: The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

Example:

A bicycle wheel has a diameter of 60 cm. How far will it travel if it makes 50 revolutions? Use $\pi = 3.14$.

The wheel will travel 50 times its circumference.

$$C = \pi d$$
$$= 3.14(60) = 188.4$$
$$\text{distance} = 50C$$
$$\approx 50(188.4) = 9420$$

It will travel about 9420 cm, or 94.2 m.

****How deep is the Well?****

The winding drum of a well is 30 cm in diameter. If it takes five and a half turns of the handle before the bucket is in the water, how deep is the well (to the nearest 10 cm)?

If the radius of the handle is 40 cm, how far has your hand travelled by the time the bucket has been brought to the top?

Solution:

Circumference of drum = $2\pi r = 2\pi(15) \approx 94.247$ cm.

In 5.5 turns, length of rope let out = 5.5 x 94.247 ≈ 518.36 cm.

Depth of well ≈ 520 cm or 5.2 m.

Circumference of hand movement = $2\pi r = 2\pi(40) \approx 251.327$ cm.

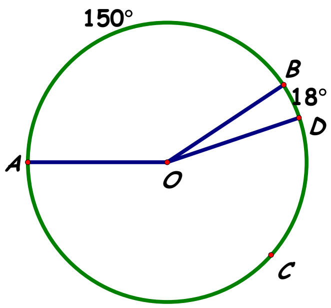
So your hand travels a total of 251.327 cm x 5.5 ≈ 1382.301 cm to pull the bucket up only 518.36 cm …

Hint: You may have to explain to pupils how an old fashioned well works!

Arc Length Corollary: In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

Simply put, this is just a fractional part of the **Circumference.**

Example: Find the arc length.



In $\odot O$ with radius 6 and $m\angle AOB = 150°$, find the lengths of $\overset{\frown}{AB}$ and $\overset{\frown}{ACD}$.

Solution:

For minor $\overset{\frown}{AB}$: Fraction: $\frac{150}{360} = \frac{5}{12}$.

Circumference: $2\pi r = 2\pi(6) = 12\pi$.

Multiply: arc length = $\frac{5}{12}(12\pi) = 5\pi$.

For major $\overset{\frown}{ACD}$: $m\overset{\frown}{ACD} = 360 - 168 = 192°$.

Fraction: $\frac{192}{360} = \frac{8}{15}$.

Multiply: arc length = $\frac{8}{15}(12\pi) = \frac{32}{5}\pi$.

Area of a Circle Theorem: The area of a circle is π times the square of the radius, or $A = \pi r^2$.

Examples:

a) The area of a circle is 40π. Find the radius of the circle in simplest form.

$$A = \pi r^2$$
$$40\pi = \pi r^2$$
$$2\sqrt{10} = r$$

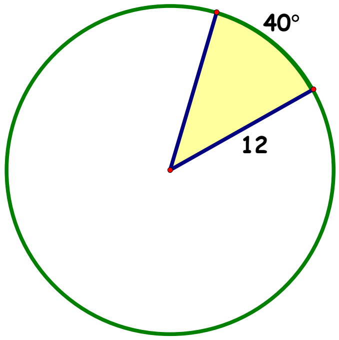
b) Find the circumference of a circle if the area is 400π.

$$A = \pi r^2 \qquad C = 2\pi r$$
$$400\pi = \pi r^2 \qquad = 2\pi(20)$$
$$20 = r. \qquad = 40\pi.$$

Area of a Sector Theorem: The ratio of the area A of a sector of a circle to the area of the circle is equal to the ratio of the measure of the intercepted arc to 360°.

Sector of a circle – a region of a circle bounded by two radii and an arc.

Example: Find the area of the shaded sector.



Solution:

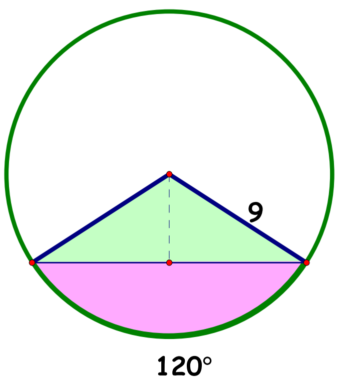
Fraction: $\frac{40}{360}=\frac{1}{9}$. Area of circle: $\pi r^2=\pi(12)^2=144\pi$.

Multiply: area of sector = $\frac{1}{9}(144\pi)=16\pi$.

Simply put, this is just a fractional part of the **Area.**

Segment of a circle – a part of the interior of a circle bounded by a chord and an arc of the circle.

Example: Find the area of the segment of the circle.



Solution:

The plan is to find the area of the sector and subtract the area of the triangle.

First find the area of sector *AOB*.

$\frac{120}{360}=\frac{1}{3} \rightarrow \pi r^2=81\pi \rightarrow$ area of sector *AOB* = $\frac{1}{3}(81\pi)=27\pi$.

If an altitude is drawn into the triangle, a 30°-60°-90° triangle is formed. The height of the triangle is $\frac{1}{2}$ the hypotenuse, or 4.5. And the longer leg is $4.5\sqrt{3}$, which is half of the base of the triangle. So, the area of the triangle is: $\frac{1}{2}(9\sqrt{3})(4.5)=20.25\sqrt{3}$. The area of the segment is $27\pi-20.25\sqrt{3}$ units^2.

****Pi Free Circles****

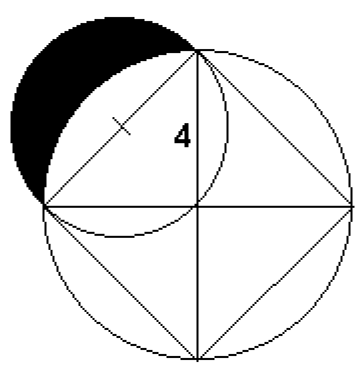
The Problem:

A square is inscribed in a circle of radius 4 cm.

A semi-circle is drawn with its center halfway along one of the sides of the square, and radius to pass through the center of the original circle.

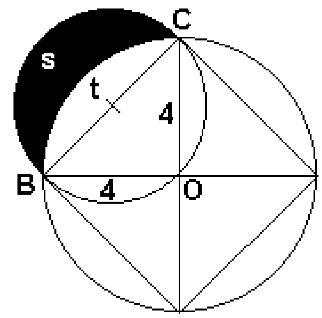
Find the area of the crescent that protrudes beyond the first circle (shown shaded).

Notes:



This is an "Area of a Circle" question with a difference – first remarked on by Hippocrates around 500 B.C.!
The segment's area turns out not to involve π.

Solution:
To find crescent area s, we subtract the segment area t from the area of the semicircle on diameter $\overline{BC}$.



To find t, we take the area of $\triangle BOC$ from the area of sector BOC of $\odot O$.

$$A_{sector} = \frac{(\text{intercepted arc})^\circ}{360^\circ}\pi r^2 = \frac{90}{360}\pi(4)^2,$$

$$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}(4)(4),$$

$$t = \frac{1}{4}\pi(16) - 8 = 4\pi - 8.$$

BC is the diameter of the semicircle,

$$BC = \sqrt{(4^2 + 4^2)} = 4\sqrt{2}.$$

$$A_{semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(2\sqrt{2}\right)^2 = 4\pi.$$

Then $s = 4\pi - (4\pi - 8) = 8\text{ cm}^2$.

Choose the initial radius to be 4 cm and pose the question of whether the "canceling" of 4π is simply a *special case*...
In fact, the whole result is a "special case".
In this case, the smaller semicircle has the same area as the larger quarter circle.

In general, if the radius of the original circle is x:

$$A_{sector} = \frac{(\text{intercepted arc})^\circ}{360^\circ}\pi r^2 = \frac{90}{360}\pi(x)^2,$$

$$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}(x)(x),$$

$$t = \frac{1}{4}\pi x^2 - \frac{1}{2}x^2 = \frac{x^2\pi}{4} - \frac{x^2}{2}.$$

BC is the diameter of the semicircle,

$$BC = \sqrt{(x^2 + x^2)} = x\sqrt{2}.$$

$$A_{semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \frac{1}{2}\pi\left(\frac{2x}{4}\right).$$

$$\text{Then } s = \frac{x^2\pi}{4} - \left(\frac{x^2\pi}{4} - \frac{x^2}{2}\right) = \frac{x^2}{2}.$$

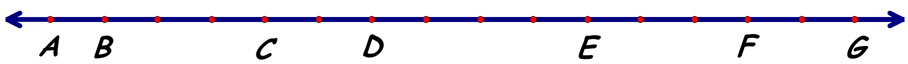
Geometric Probability – using the principles of length and area to find the probability of an event.

a) Probability and Length – Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is as follows: $P\left(\text{Point } K \text{ is on } \overline{CD}\right) = \dfrac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$.

b) Probability and Area – Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is as follows: $P\left(\text{Point } K \text{ is in region } M\right) = \dfrac{\text{Area of } M}{\text{Area of } J}$

Examples:

a) A point is picked at random on $\overline{AG}$. What is the probability that X is on:



a) $\overline{AC}$ $\dfrac{4}{15}$

b) $\overline{CD}$ $\dfrac{2}{15}$

c) $\overline{CE}$ $\dfrac{6}{15} = \dfrac{2}{5}$

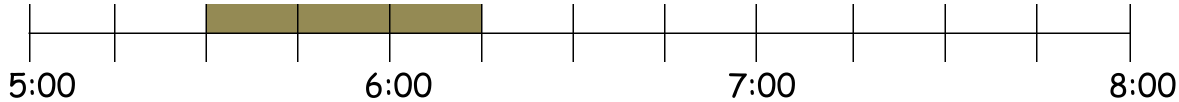
d) $\overline{DF}$ $\dfrac{7}{15}$

e) $\overline{AG}$ $\dfrac{15}{15} = 1$

f) $\overline{FG}$ $\dfrac{2}{15}$

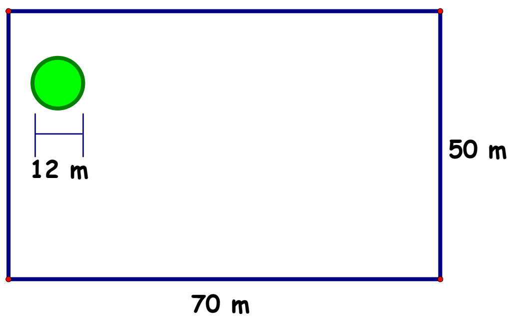
b) A friend promises to call you at home sometime between 5 PM and 8 PM. At 5:30, you must leave your house unexpectedly for 45 minutes. What is the probability you miss the call?

Think of a timeline. The shaded region represents the interval you are not at home. Between 5 PM and 8 PM you are not home for three-quarters of an hour.



P(you will miss the call) = $\dfrac{\textit{length of shaded segment}}{\textit{length of the whole segment}} = \dfrac{\frac{3}{4}}{3} = \dfrac{3}{4} \bullet \dfrac{1}{3} = \dfrac{1}{4}$.

c) A parachutist jumps from an airplane and lands in the rectangular field shown. What is the probability that the parachutist avoids the tree represented by a circle in the diagram? (Assume that the person is unable to control the landing point.)



Probability of avoiding the tree = $\dfrac{\text{area of rectangle - area of shaded region}}{\text{area of rectangle}}$.

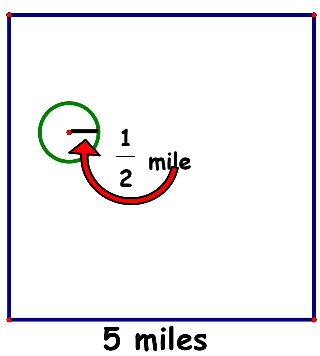
Area of rectangle = $70 \bullet 50 = 3500\,m^2$.

Area of shaded region = $\pi r^2 \approx 3.14 \bullet 6^2 \approx 113\,m^2$.

$$\frac{\text{area of rectangle - area of shaded region}}{\text{area of rectangle}} = \frac{3500 - 113}{3500} \approx 0.97.$$

d) A ship has sunk in the ocean in a square region 5 miles on a side. A salvage vessel anchors at a random spot in this square. Divers search half a mile in all directions from the point on the ocean floor directly below the vessel.

i. What is the approximate probability that they will locate the sunken ship at the first place they anchor?



P(finding the ship on 1^{st} try) = $\dfrac{\text{area of circle}}{\text{area of square}} = \dfrac{.25\pi}{25} \approx .03$ *or* 3%.

ii. What is the approximate probability they won't find the ship on the first try?

P(not finding the ship on the 1^{st} try) = 100 – 3 = 97%.

iii. What is the approximate probability that they locate the sunken ship in five tries? (Assume that the tries do not overlap.)

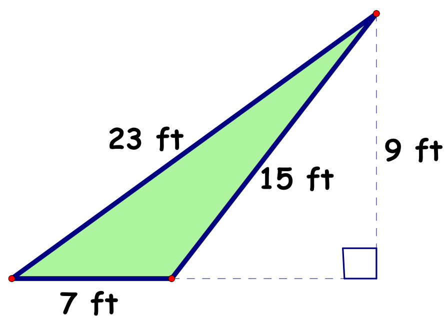
P(finding the ship on 5 tries) = $\dfrac{5(\text{area of circle})}{\text{area of square}} = \dfrac{1.25\pi}{25} \approx .16$ *or* 16%.

Keep looking!

Syllabus Objective: 8.3 - The student will solve real world problems of perimeter and area.

Examples:

a) Andrew needs enough mulch to cover the triangular garden shown and enough paving stones to border it. If one bag of mulch covers 12 square feet and one paving stone provides a 4-inch border, how many bags of mulch and how many stones does he need to buy?



Find the perimeter of the garden.

Perimeter of the garden = 23 + 15 + 7 or 45 ft.

Find the area of the garden.

$A = \frac{1}{2}bh = \frac{1}{2}(7)(9)$ or $31.5\ \text{ft}^2$.

Use unit analysis to determine how many of each item are needed.

Bags of Mulch:

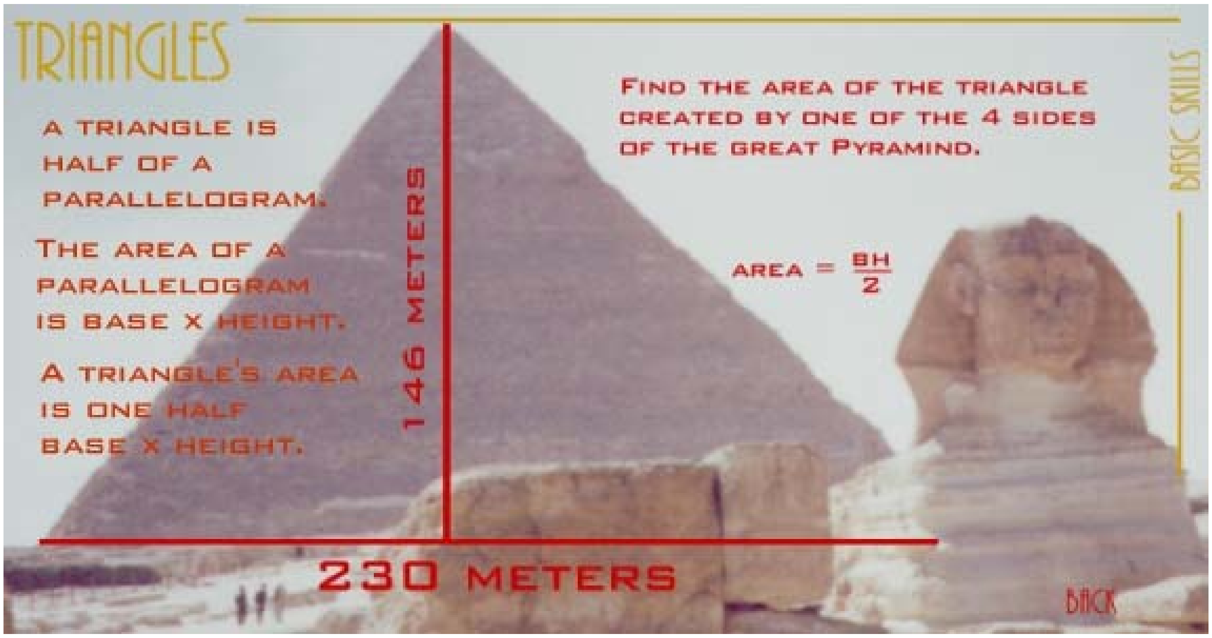
$31.5\,\text{ft}^2 \bullet \frac{1\,\text{bag}}{12\,\text{ft}^2} = 2.625\,\text{bags}.$

Paving Stones:

$45\,\text{ft} \bullet \frac{12\,\text{in.}}{1\,\text{ft}} \bullet \frac{1\,\text{stone}}{4\,\text{in.}} = 135\,\text{stones}.$

Round the number of bags up so there is enough mulch. He will need 3 bags of mulch and 135 paving stones.

b) Some scholars believe the pyramids were once painted white. If a gallon of white wash can cover 44 square meters of surface, how many gallons were needed to paint the entire pyramid?



Although this question is asking for the lateral area of a pyramid, it is approached as the area of the four triangular sides of a pyramid and provides linkage to future solid topics.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(230)(146)$$
$$= 16790 \text{ m}^2.$$

$$4(16790) = 67160$$

$67160 \div 44 = 1526.\overline{36}$ gallons of white wash.

Round up to 1527 gallons.

c) Find the circumference of the ditch and the area it encloses.

$$r = \frac{d}{2} = \frac{104}{2} = 52 \text{ m.}$$

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(52) = 104\pi \approx 326.7 \text{ m.} \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(52)^2 = 2704\pi \approx 8494.9 \text{ m}^2. \end{aligned}$$

d)

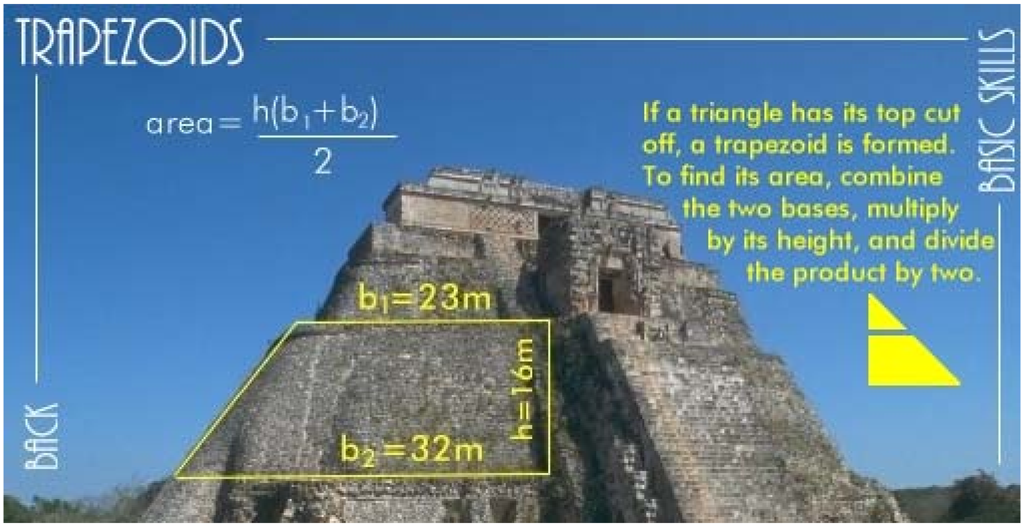
$$\begin{aligned} P &= 2(w + l) \\ &= 2(8 + 12) = 40 \end{aligned}$$

Total perimeter: $7(40) = 280$ m.

$$\begin{aligned} A &= wl \\ &= (8)(12) = 96 \end{aligned}$$

Total area: $7(96) = 672$ m^2.

e) The Pyramid of the Magician at Uxmal in Mexico is made of stones. If the average size of the rectangular stones is 60cm by 26cm, about how many stones cover the surface of the area within the yellow trapezoid?



Area of each stone:

$A = bh = (60)(26) = 1560 \text{ cm}^2.$

$A = \frac{1}{2}h(b_1 + b_2)$

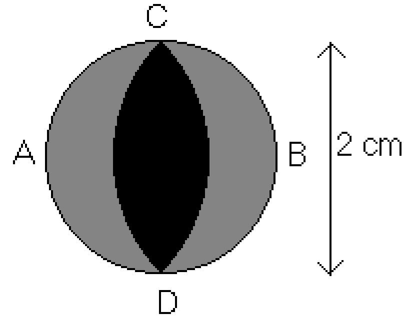
$= \frac{1}{2}(16)(23+32) = 440 \text{ m} = 44000 \text{ cm}^2.$

Number of stones:

$44000 \div 1560 \approx 28.2.$

Snake Eyes

The King Cobra is the world's largest poisonous snake. It can grow to over 5 meters long and has large bronze-colored eyes, with a black central 'slit'. The shape of the eyes is shown below.



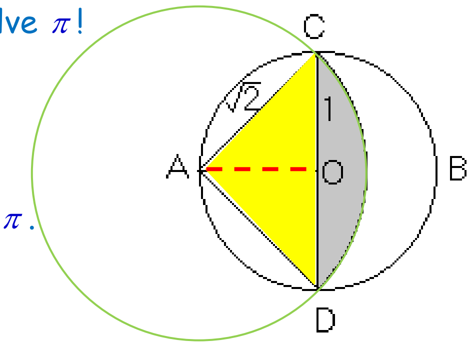
Take the arcs drawn between C and D as having centers A and B.

The Problem:

Find the total area of the lightly shaded region (the bronze part of a Cobra's eye).

Solution:

Surprisingly, the area concerned does not involve π!



Radius of sector *CAD:* $\sqrt{2}$. (sector of $\odot A$.)

Angle of sector *CAD:* 90°.

Hence area of sector *CAD:* $\frac{1}{4}\pi\left(\sqrt{2}\right)^2 = \frac{1}{2}\pi$.

Area of ΔCAD: 1

Hence shaded area: $\frac{1}{2}\pi - 1$.

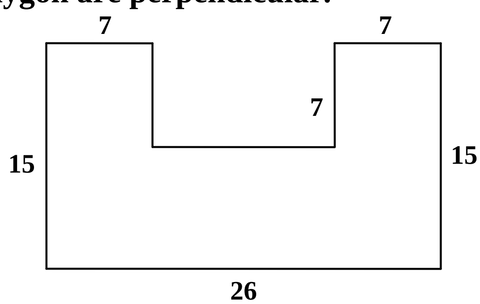
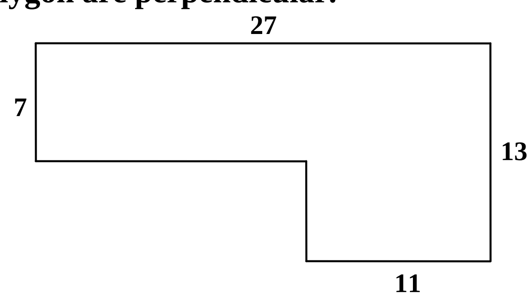
Area of the lightly shaded region: $\pi\left(1\right)^2 - 2\left(\frac{1}{2}\pi - 1\right)$

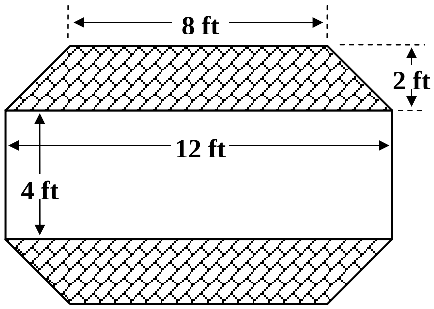
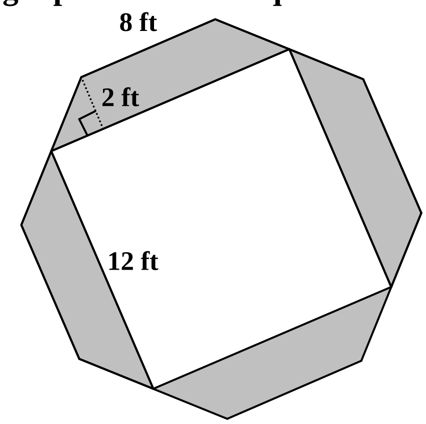
= **2 cm².**

This is rather a nice change for those of us accustomed to π cropping up whenever circles are involved!

This unit is designed to follow the Nevada State Standards for Geometry, CCSD syllabus and benchmark calendar. It loosely correlates to Chapter 11 of McDougal Littell *Geometry* © 2004, sections 1.7, 6.7, 11.2 – 11.5. The following questions were taken from the 2nd semester common assessment practice and operational exams for 2008-2009 and would apply to this unit.

Multiple Choice

#	Practice Exam (08-09)	Operational Exam (08-09)
1.	**A tire has a radius of 15 inches. What is the approximate circumference, in inches, of the tire?** **A.** 47 in. **B.** 94 in. **C.** 188 in. **D.** 707 in.	**A circular steering wheel has a radius of 7 inches. What is the approximate circumference of the steering wheel in inches?** **A.** 22 inches **B.** 44 inches **C.** 88 inches **D.** 154 inches
2.	**In the figure below, adjacent sides of the polygon are perpendicular.**  **What is the perimeter of the figure?** **A.** 77 **B.** 82 **C.** 89 **D.** 96	**In the figure below, the adjacent sides of the polygon are perpendicular.**  **What is the perimeter of the figure?** **A.** 58 **B.** 77 **C.** 80 **D.** 91
3.	**The length of a rectangular patio is 32 feet. Its area is 800 square feet. What is the perimeter of the patio in feet?** **A.** 25 ft **B.** 57 ft **C.** 114 ft **D.** 368 ft	**The length of a rectangular garden is 60 feet. Its area is 2400 square feet. What is the perimeter of the garden in feet?** **A.** 100 feet **B.** 200 feet **C.** 2440 feet **D.** 2460 feet

4.	**A rectangular garden is to be edged with decorative brick as shown by the shaded region in the figure. The flower garden is 4 feet by 12 feet. The trapezoids are 2 feet high.**  **What is the area of the decorative edge (the shaded region) in square feet?** **A.** 20 ft^2 **B.** 26 ft^2 **C.** 40 ft^2 **D.** 48 ft^2	**A square flower garden is to be edged with decorative brick as shown by the shaded regions in the figure.** **The flower garden is 12 feet by 12 feet. The shaded regions are 2 feet high and the outer edges parallel to the square are 8 feet long.**  **What is the area of the decorative edge (the shaded region) in square feet?** **A.** 22 ft^2 **B.** 80 ft^2 **C.** 144 ft^2 **D.** 192 ft^2

Free Response

Practice Exam (08-09)
3. Find the area of a regular hexagon with an apothem of 9 centimeters. Give answer in simplified radical form.

Operational Exam (08-09)
3. Find the area of a regular hexagon with a side of 6 centimeters. Give the answer in simplified radical form.