Math Audit Team
Regional Professional Development Program
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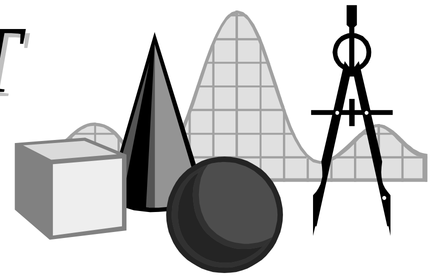
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

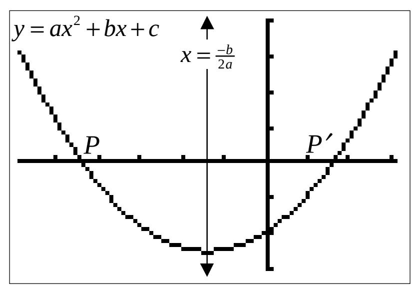


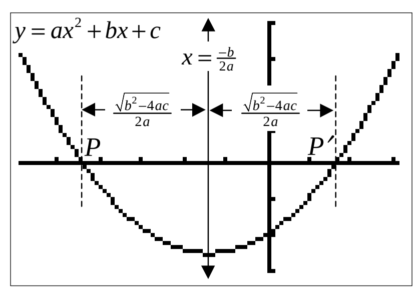
"Memorize this!" Those famous words—or infamous where students are concerned—are heard most often when the quadratic formula is presented. Students dutifully commit the recipe to memory and even understand that it gives solutions to quadratic equations. Yet, few students understand its connection to characteristics of quadratic functions and parabolas. In this issue of *Take It to the MAT*, we will look at a few of those connections.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{where } ax^2 + bx + c = 0,\ a \neq 0$$

Students must have the understanding that the quadratic formula not only gives the solutions to a quadratic equation, but since the graph of a quadratic is a parabola, that the quadratic formula gives the x-intercepts of $y = ax^2 + bx + c$. Often students can solve quadratic equations with the quadratic formula, factoring, completing the square, etc., and can find x-intercepts on graphs of parabolas by using a graphing calculator or making a table, but sometimes they do not see the connection between solutions and x-intercepts. It is up to teachers to help them see the link.

To build the link between solutions and intercepts, first reinforce the symmetric nature of parabolas. One of the most splendid features of a parabola is its axial symmetry, its symmetry with respect to a line. The line is called the *axis of symmetry* and students easily learn that its equation is $x = \frac{-b}{2a}$. Thus, for any point P on the parabola there is a point P' that is the reflection of P about the line of symmetry. Isn't one of the x-intercepts a point on the parabola, and doesn't it thus have a reflected point which is also an x-intercept?





Since the two x-intercepts are symmetric with respect to the line $x = \frac{-b}{2a}$, they are the same distance from it. By rewriting the quadratic formula in a little different format, this can be easily seen; $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. Or,
x = (location of axis of symmetry) ± (some distance).

Now the formula is less "magical"—it really does have some meaning. There is a logical connection between the formula and the graph.

As an exercise, consider the following equation: $y = a(x - p)(x - p')$. Where are the x-intercepts? What is the equation of the axis of symmetry? What is the distance from the axis of symmetry to the x-intercepts? What are the solutions if $a(x - p)(x - p') = 0$ is run through the quadratic formula?