TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Southern Nevada Regional Professional Development Program
March 2005 — Middle School Edition

www.rpdp.net

Take It to the MAT continues its examination of errors in language with respect to proportional reasoning. In the February 2005 edition, the discussion focused on *percent of*, *percent more than*, and *percent less than*. This time, we will look at *absolute* differences in percentage as compared to *relative* differences.

Let's start this time with a real-world situation. Consider a state where sales taxes are presently at 5%. A legislator wants to raise the sales tax rate to 6%. The lawmaker states that the tax increase amounts to only 1 percent. Not a large increase at all. A taxpayer watchdog group claims that this is a 20% increase in taxes, a huge jump in the burden on taxpayers. Who is correct?

The legislator is looking at the difference in *absolute* terms. The *absolute difference* between 6% and 5% is 1 percent. The representative's statement that the increase is "only 1 percent" is misleading, however. What the lawmaker should say is that taxes will increase by one *percentage point.* An increase of one percentage point is quite different from an increase of one percent.

The watchdog group is describing the difference in *relative* terms. The *relative difference* between 6% and 5% is 20 percent—not one percent. That is, the *increase* of one percent relative to the original condition of five percent is 20%. One is twenty percent of five. Relative changes are always based on what the initial condition is.

Look at it another way. If we were to purchase an item that costs \$200, the 5% sales tax on the item would be \$10.00. At a 6% rate, the tax would be \$12.00. There is an *absolute* \$2.00 difference between the taxes paid, but the *relative* increase is \$2.00 out of \$10.00, or 20%.

As a formula: $\textit{relative}\,\text{change} = \dfrac{\text{final condition} - \text{initial condition}}{\text{initial condition}} = \dfrac{\textit{absolute}\,\text{change}}{\text{initial condition}}$.

In our case: $\textit{relative}\,\text{change} = \dfrac{6\% - 5\%}{5\%} = \dfrac{1\%}{5\%} = 0.20 = 20\%$.

Note that if the final condition is less than the initial condition, the result will be negative, thus indicating a relative *decrease*. For example, consider another state where sales tax rates are at 5% and legislation is proposed to lower it to 4%. (Unrealistic, yes, but let's go with it for the sake of doing the math.) What is the relative decrease in taxes?

$$\textit{relative}\,\text{change} = \frac{4\% - 5\%}{5\%} = \frac{-1\%}{5\%} = -0.20 = -20\%.$$

The relative change is –20%, or a 20 percent *decrease* in taxes. Think of it as having to pay only \$8.00 tax on a \$200 item versus \$10.00.

The method to compute relative change works for any before/after scenario, not just percentage tax rates. It can be applied to growth of houseplants or to the time it takes to complete a task. Percentages are presented here because students often get confused when comparing percentages to other percentages.