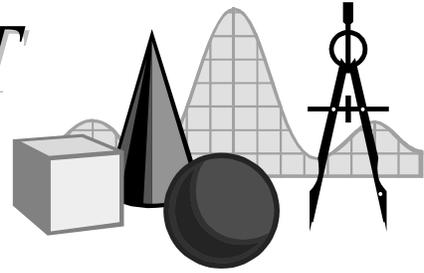


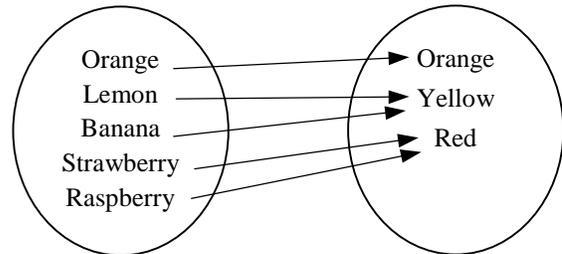
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

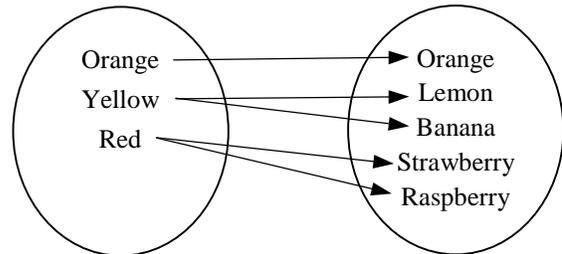
Math Audit Team
Regional Professional Development Program
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We start this third *Take It to the MAT* in a series about relations and functions with a modified example from the last issue. A pairing is done of the set of fruits {Orange, Lemon, Strawberry, Banana, Raspberry} and the set of colors {Red, Orange, Yellow}. We can show that the pairing of fruits to their color is a function because for *each* element in the first set is paired with *exactly one* element in the second.



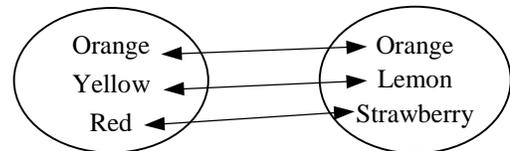
Let us now reverse the roles of the sets, pairing each color with the fruit for which it is the general color. The new pairing is a relation, but not a function because Red and Yellow both pair with two fruits. (Some dictionaries define this situation as a *multiple-valued function*. Interestingly, these dictionaries state that a multiple-valued “function” is not a function under our definition.)



The point behind reversing the sets that contain the domain and the range is to begin to create *inverse functions*, that is: *If a function f pairs each element of a set X with a unique element of set Y , the inverse of f , f^{-1} , pairs each element of set Y with a unique element of set X .*

Many functions do not have inverses that are also functions because several elements of X pair with the same element of Y . The function $y = x^2$ immediately comes to mind. Two conditions must be met for there to be an inverse function: (1) every member of X must pair with exactly one member of Y , and (b) every member of Y must pair with exactly one member of X . Thus, there must be a *one-to-one correspondence* between the sets X and Y . What does that suggest about the size of the sets?

For a function to have an inverse it must be *one-to-one*. If we were to drop Banana and Raspberry from the fruit set in our examples, we then have a one-to-one function no matter which way we do the pairing between the sets.



In the case of the function $y = x^2$, the inverse *relation* would be $x = \pm\sqrt{y}$. If we were to limit the domain of x to only non-negative real numbers ($x \geq 0$), then we would have a one-to-one function and the inverse *function* would be $x = \sqrt{y}$.

Drawing some conclusions: (1) Every relation is not a function. (2) Every function has an inverse which may or may not itself be a function. (3) For a function to have an inverse which is also a function it must be one-to-one.