Syllabus Objective: 1.2 – The student will solve problems using the algebra of functions.

Modeling a Function:

- Numerical (data table)
- Algebraic (equation)
- Graphical

Using Numerical Values:
Look for a **common difference**.

- If the *first* difference is constant, the function is *linear*.
- If the *second* difference is constant, the function is *quadratic*.
- If the *third* difference is constant, the function is *cubic*.

Ex1: Determine the type of function using the common difference. Graph the function and write an equation for the function.

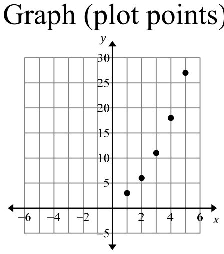
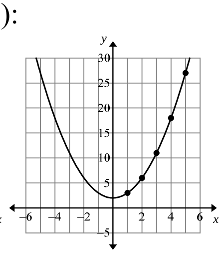
x	1	2	3	4	5
y	3	6	11	18	27

First difference: 3 5 7 9

Second difference: 2 2 2

Second difference is constant, so it is quadratic.

Graph (plot points):

Because of the symmetry in a quadratic function, we can draw the entire function.

Write the equation: Extending the table, we can find the *y*-intercept. $(0,2)$

So the equation of the function is $\boxed{y = x^2 + 2}$

x-Intercept: the value of *x* where the graph intersects the *x*-axis; also called a **zero** or **root**

<u>Using an Algebraic Model:</u>

Ex2: Find the zeros of the function algebraically and graph. $f(x) = 2x^3 - 2x^2 - 8x + 8$

Factor by grouping: $f(x) = 2x^2(x-1) - 8(x-1) \Rightarrow f(x) = (2x^2 - 8)(x-1)$

Use the zero product property (A product of real numbers is zero if and only if at least one of the factors in the product is zero.):

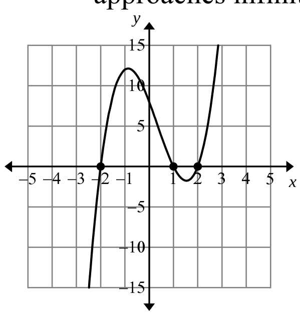
$2x^2 - 8 = 0$ $\quad$ $x - 1 = 0$ $\quad$ Zeros: $\boxed{\{-2, 1, 2\}}$

$x = -2, 2$ $\quad$ $x = 1$

To sketch the graph, we can use the zeros and end behavior.

Recall: In a cubic function, if *a* is positive, as $x \to -\infty$, $f(x) \to -\infty$; and as $x \to \infty$, $f(x) \to \infty$.

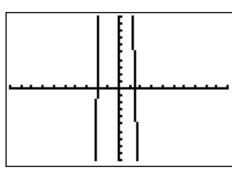
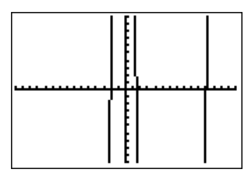
Read As: "As *x* approaches negative infinity, *f* of *x* approaches negative infinity. As *x* approaches infinity, *f* of *x* approaches infinity."

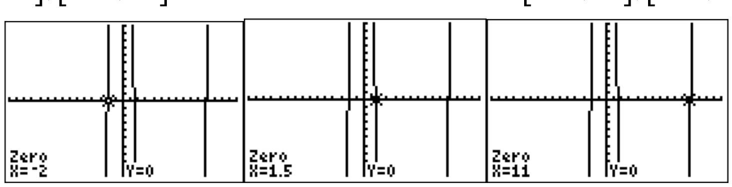


<u>Using a Graph</u>: Watch for "hidden" behavior!

Ex3: Solve the equation $2x^3 - 21x^2 - 17x + 66 = 0$ graphically.

Graph on the calculator (Zoom Standard). Because this is a cubic function, we know by the end behavior that it will have one more zero. Extend the window on the *x*-axis to −15 to 15.

Window: $[-10, 10], [-10, 10]$  Window: $[-15, 15], [-10, 10]$ 



Solutions: $\boxed{x = -2, 1.5, 11}$

Ex4: Graph a scatter plot for gas prices and find an algebraic model. When will gas prices hit \$8.00/gallon?


CRAIG L. MORAN/REVIEW-JOURNAL

Jetta Lathan, left, prepares to board a bus Tuesday on Maryland Parkway. Lathan, who has ridden the bus since her car was wrecked 18 months ago, said more routes are needed for residents who want to commute.

More people taken for ride

Number of bus passengers climbs steadily along with cost of filling gasoline tanks

Gasoline prices
Per gallon of regular unleaded

2002	\$1.61
2003	\$1.85
2004	\$2.15
2005	\$2.58
2006	\$2.77
2007	\$3.12
2008	\$4.13

REVIEW-JOURNAL
SOURCE: AAA NEVADA

Las Vegas Valley
Population growth

2002	1.52 mil.
2003	1.58 mil.
2004	1.69 mil.
2005	1.75 mil.
2006	1.86 mil.
2007	1.93 mil.
2008	NA

REVIEW-JOURNAL
SOURCE: CLARK COUNTY

Transit ridership

2002	45.3 mil.
2003	48.4 mil.
2004	51.6 mil.
2005	55.6 mil.
2006	61.0 mil.
2007	63.8 mil.
2008	65.7 mil.*

*Projected, based on expected 3 percent growth
REVIEW-JOURNAL
SOURCE: RTC

By SCOTT WYLAND
REVIEW-JOURNAL

His gold T-shirt reflects the torrid midday sun as he stands at the edge of the curb, peering down the street in the hope of spotting a bus in the oncoming traffic.

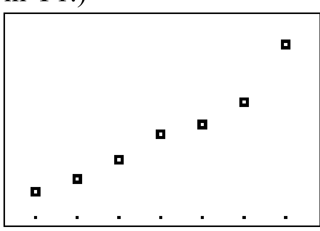
Joe Cuff, 54, lost his job as a federal insurance inspector, then lost his car a year ago when he could no longer afford it.

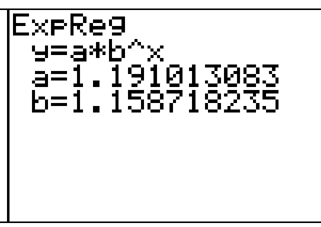
He is happy to ride the bus and forgo buying another car, especially now that gasoline prices have surged past \$4 a gallon and are creeping ever upward.

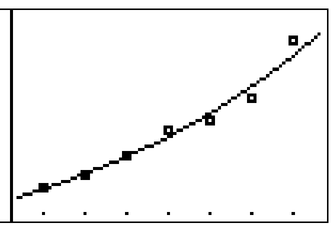
► SEE BUSES PAGE 4A
\$40 monthly pass looks more inviting

ON THE WEB To plan your trip from home to work or any other place in Las Vegas, go to the RTC Web site
► http://ttpweb.rtcsouthernnevada.com/

The year is the independent variable *x*. We will label 2002 as $x = 2$ and create a scatter plot. This appears to be an exponential model, so we will use ExpReg to find an algebraic model. (Note: Store this in Y1.)

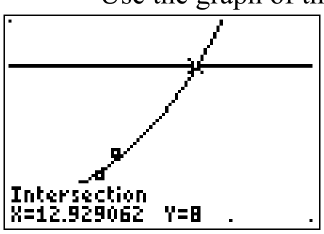






$y = 1.191(1.159)^x$

Use the graph of the algebraic model to estimate when gas prices will hit \$8.00.



Gas prices will hit \$8.00 close to the year 2013.

▽ Note: If *a* is a real number that solves the equation $f(x) = 0$, then these three statements are equivalent.

- *a* is a **root** (solution) of the equation $f(x) = 0$.
- *a* is a **zero** of $y = f(x)$.
- *a* is an ***x*-intercept** of the graph of $y = f(x)$.

<u>You Try</u>: Solve the equation algebraically and graphically. $(x+2)^2 = 9$

<u>QOD</u>: What are two ways to solve an equation graphically?

Syllabus Objectives: 1.3 – The student will determine the domain and range of given functions and relations. 2.7 – The student will analyze the graph of a function for continuity. 2.5 – The student will describe the symmetries for the graph of a given relation. 2.6 – The student will compare values of extrema for a given relation. 2.4 – The student will find the vertical and horizontal asymptotes of the graph of a given relation.

Function: a rule that assigns every element in the domain to a **unique** element in the range

Function Notation: $y = f(x)$; Read "*y* equals *f* of *x*"

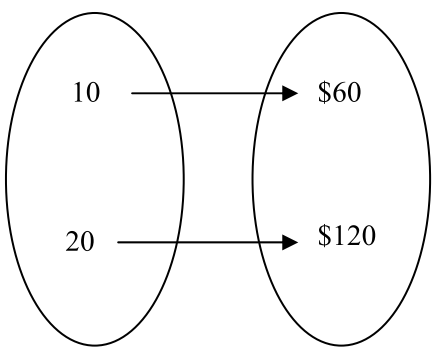
Domain: the values of the independent variable (*x*)

Range: the values of the dependent variable (*y*)

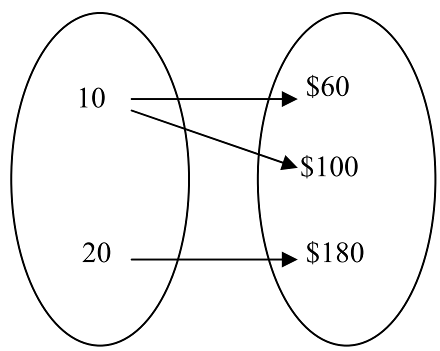
Functions can be represented using a mapping diagram.

Ex1: Which of these mapping diagrams represents a function, where the domain represents the number of entrees ordered, and the range represents the amount paid?

Taco Bell



P.F. Changs



Function: for every input, there is exactly one output

Not a Function

Think About It: What difference between the two restaurants would be the reason for one being a function and the other not? (tipping)

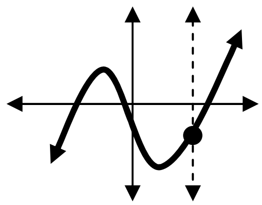
Using a **graph** to determine if a relation is a function:

Graphically, the vertical line test will tell if *x* repeats.

Vertical Line Test: If any vertical line intersects a graph at **more than one** point, then the graph is **NOT** a function.

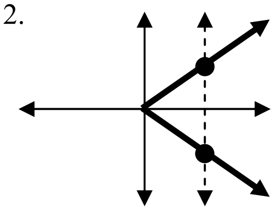
Ex2: Determine if the graphs are functions and explain why or why not.

1.



Function: Vertical line does not intersect the graph at more than one point.

2.



Not a Function: Vertical line intersects the graph at more than one point.

Ex3: A function has the property that $f(3x-1)=x^2+x+1$ for all real numbers *x*. What is $f(5)$?

Find when the input, $3x-1$, equals 5: $3x-1=5 \Rightarrow x=2$

Evaluate the function when $x=2$: $f(5)=f(3(2)-1)=(2)^2+2+1=\boxed{7}$

Determine domain and range from an equation:

Ex4: Find the domain and range of the function. $f(x)=\dfrac{\sqrt{x}}{x-2}$

Domain: The radicand cannot be negative, so $x \geq 0$. The denominator cannot equal zero, so $x \neq 2$.

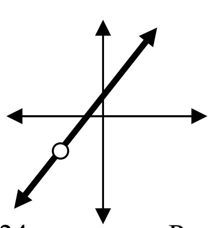
Range: The numerator can only be positive, and the denominator can be all real numbers. So the range is all real numbers

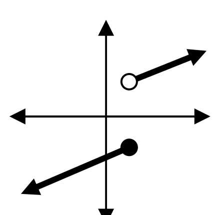
Write the answers in interval notation. Domain: $\boxed{[0,2)\cup(2,\infty]}$ Range: $\boxed{(-\infty,\infty)}$

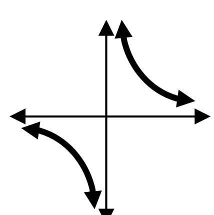
Continuity: a graph is **continuous** if you can sketch the graph without lifting your pencil

Types of Discontinuity:

1. Removable (hole)
2. Jump
3. Infinite





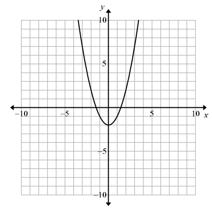


<u>Symmetry</u>: a graph of a function can be **even**, **odd**, or neither

<u>Even Function</u>: a function that is symmetric about the *y*-axis

If a function is **even**, then $f(-x) = f(x)$.

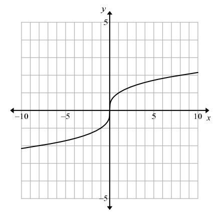
Example of an even function: $y = x^2 - 2$



<u>Odd Function</u>: a function that is symmetric with respect to the origin

If a function is **odd**, then $f(-x) = -f(x)$.

Example of an odd function: $y = \sqrt[3]{x}$



Ex5: Show whether the function is odd, even or neither. $y = x^3 - 2x$

Find $f(-x)$. $f(-x) = (-x)^3 - 2(-x) \Rightarrow f(-x) = -x^3 + 2x = -(x^3 - 2x)$

$f(-x) = -f(x)$, so the function is ODD.

Ex6: Show whether the function is odd, even or neither. $g(x) = |x| + 3$

Find $f(-x)$. $f(-x) = |-x| + 3 \Rightarrow f(-x) = |x| + 3$

$f(-x) = f(x)$, so the function is EVEN.

<u>Relative Extrema</u>: the **maxima** or **minima** of a function in a local area

<u>Absolute Extrema</u>: the **maximum** or **minimum** value of a function in its domain

<u>Increasing Function</u>: as the *x*-values increase, the *y*-values increase

<u>Decreasing Function</u>: as the *x*-values increase, the *y*-values decrease

<u>Constant Function</u>: as the *x*-values increase, the *y*-values do not change

<u>Boundedness</u>: A function is *bounded below* if there is some number *b* that is less than or equal to every number in the range of the function. A function is *bounded above* if there is some number *B* that is greater than or equal to every number in the range of the function. A function is *bounded* if it is bounded both above and below.

Asymptotes:

- *Horizontal Asymptote*

  The line $y = b$, if $f(x)$ approaches *b* as *x* approaches $+\infty$ or $-\infty$.

  $y = \lim_{x\to-\infty} f(x)$ and $y = \lim_{x\to\infty} f(x)$

- *Vertical Asymptote*

  The line $x = a$, if $f(x)$ approaches $+\infty$ or $-\infty$ as *x* approaches *a* from either direction.

Recall:

Limit: the value that $f(x)$ **approaches** as *x* **approaches** a given value

Notation: $\lim_{x\to a} f(x)$ "The limit as *x* approaches *a* of $f(x)$." (From both sides.)

Calculating a Limit (as *x* approaches positive or negative infinity): divide every term by the largest power of *x*

Ex7: Find the following for the function $f(x) = \dfrac{x+1}{x-2}$. Then graph the function.

Domain, range, continuity, increasing/decreasing, symmetry, boundedness, extrema, asymptotes

Domain: The denominator equals zero when $x = 2$, so the domain is $\boxed{(-\infty,2)\cup(2,\infty)}$.

Range: The function can take on all real values, so the range is $\boxed{\text{all Reals}}$.

Continuity: The function has an infinite discontinuity at $x = 2$.

Increasing/Decreasing: Always decreasing

Symmetry: $f(-x) = \dfrac{-x+1}{-x-2}$, so the function is neither even nor odd.

Boundedness: The function is not bounded above or below.

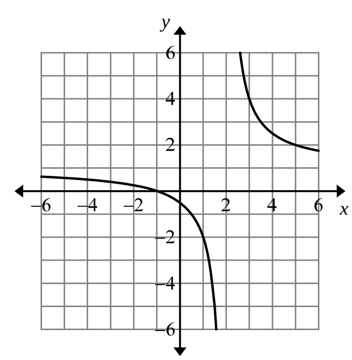
Extrema: None

Vertical Asymptote: $x = 2$

Horizontal Asymptote:

$$\lim_{x\to-\infty}\frac{x+1}{x-2} = \lim_{x\to-\infty}\frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{2}{x}} = \lim_{x\to-\infty}\frac{1+\frac{1}{x}}{1-\frac{2}{x}} = \frac{1+0}{1-0} = 1$$

$$\lim_{x\to\infty}\frac{x+1}{x-2} = \lim_{x\to\infty}\frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{2}{x}} = \lim_{x\to\infty}\frac{1+\frac{1}{x}}{1-\frac{2}{x}} = \frac{1+0}{1-0} = 1$$



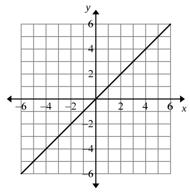
$y = 1$

You Try: Graph the function. Identify the domain, range, continuity, increasing/decreasing, symmetry, boundedness, extrema, and asymptotes. $f(x)=\dfrac{x^2-9}{x+3}$

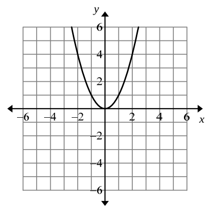
QOD: Identify a rule for finding the horizontal asymptotes of a function. Hint: Use the degrees of the numerator and denominator.

Syllabus Objective: 2.9 – The student will sketch the graph of a polynomial, radical, or rational function.
Teacher Note: For each of the 12 basic functions, have students identify the domain, range, increasing/decreasing, symmetry, boundedness, extrema, and asymptotes.

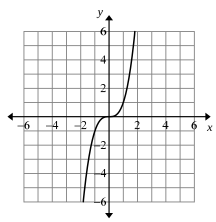
1. Identity Function $y = x$



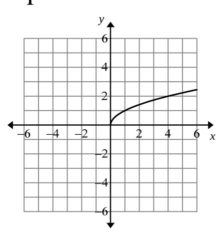
2. Squaring Function $y = x^2$



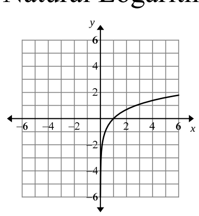
3. Cubing Function $y = x^3$



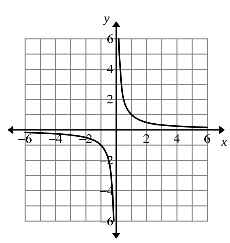
4. Square Root Function $y = \sqrt{x}$



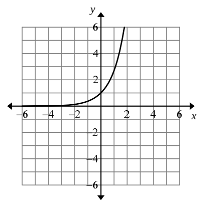
5. Natural Logarithm Function $y = \ln x$



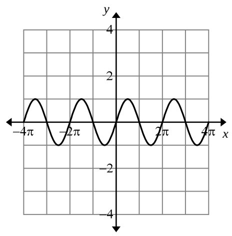
6. Reciprocal Function $y = \dfrac{1}{x}$



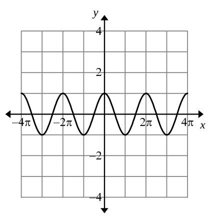
7. Exponential Function $y = e^x$



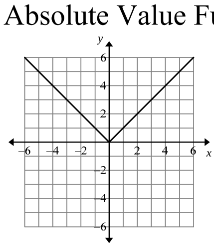
8. Sine Function $y = \sin x$



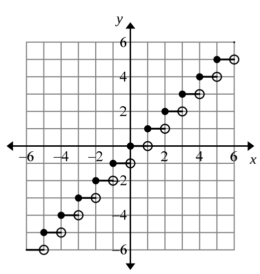
9. Cosine Function $y = \cos x$



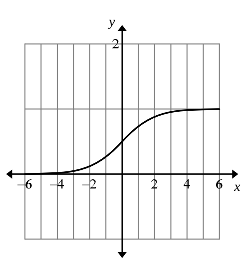
10. Absolute Value Function



11. Greatest Integer Function $y = \text{int}(x) = [\![x]\!]$ (Step Function)



12. Logistic Function $y = \dfrac{1}{1 + e^{-x}}$



Ex1: The greatest integer function is evaluated by finding the greatest integer less than or equal to the number. Evaluate the following if $f(x) = [\![x]\!]$.

a) $f\left(\frac{1}{3}\right)$ The greatest integer less than or equal to $\frac{1}{3}$ is $\boxed{0}$.

b) $f(\pi)$ The greatest integer less than or equal to π is $\boxed{3}$.

c) $f(-3)$ The greatest integer less than or equal to -3 is $\boxed{-3}$.

d) $f(-9.2)$ The greatest integer less than or equal to -9.2 is $\boxed{-10}$.

Discussing the Twelve Basic Functions:

- **Domain**: Nine of the basic functions have domain the set of all real numbers.

  The Reciprocal Function $y = \frac{1}{x}$ has a domain of all real numbers except $x = 0$.

  The Square Root Function has a domain of $[0, \infty)$.

  The Natural Logarithm Function has a domain of $(0, \infty)$.

- **Continuity**: Eleven of the basic functions are continuous on their entire domain.

  The Greatest Integer Function $y = [\![x]\!]$ has jump discontinuities at every integer value.

  ▽Note: The Reciprocal Function $y = \frac{1}{x}$ has an infinite discontinuity at $x = 0$. However, it is still a continuous function because $x = 0$ is not in its domain.

- **Boundedness**: Three of the basic functions are bounded.
  The Sine and Cosine Functions are bounded above at 1 and below at −1.
  The Logistic Function is bounded above at 1 and below at 0.

- **Symmetry**: Three of the basic functions are even. Four of the basic functions are odd.

  The Squaring Function $f(x) = x^2$ is even because $f(-x) = (-x)^2 = x^2$.

  The Cosine Function $f(x) = \cos x$ is even because $f(-x) = \cos(-x) = \cos(x)$.

  The Absolute Value Function $f(x) = |x|$ is even because $f(-x) = |-x| = |x|$.

  The Identity Function $f(x) = x$ is odd because $f(-x) = -x$.

  The Cubing Function $f(x) = x^3$ is odd because $f(-x) = (-x)^3 = -x^3$.

  The Reciprocal Function $f(x) = \frac{1}{x}$ is odd because $f(-x) = \frac{1}{(-x)} = -\frac{1}{x}$.

  The Sine Function $f(x) = \sin x$ is odd because $f(-x) = \sin(-x) = -\sin x$.

- **Asymptotes**: Two of the basic functions have vertical asymptotes at $x = 0$. Three of the basic functions have horizontal asymptotes at $y = 0$.

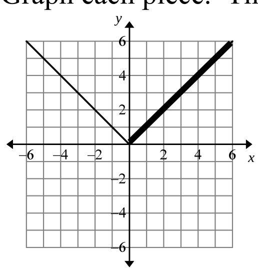
The Reciprocal Function $y = \frac{1}{x}$ and the Natural Logarithm Function $y = \ln x$ have vertical asymptotes at $x = 0$.

The Reciprocal Function $y = \frac{1}{x}$, the Exponential Function $y = e^x$, and the Logistic Function $y = \frac{1}{1+e^{-x}}$ have horizontal asymptotes at $y = 0$.

<u>Piecewise Function</u>: A function whose rule includes more than one formula. The formula for each piece of the function is applied to certain values of the domain, as specified in the definition of the function.

Ex2: Graph the piecewise function. $f(x) = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

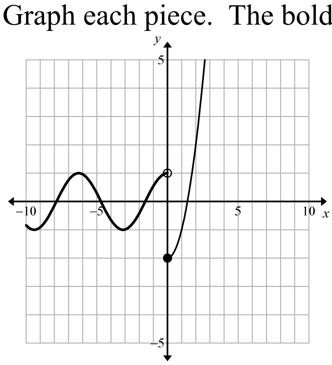
Graph each piece. The bolded portion is the first "piece".



Note: This is the Absolute Value Function!

Ex3: Graph the piecewise function. $\begin{cases} \cos x & x < 0 \\ x^2 - 2 & x \ge 0 \end{cases}$

Graph each piece. The bolded portion is the first "piece".



Note: There is a jump discontinuity at $x = 0$.

<u>You Try</u>: Graph the piecewise function. $\begin{cases} -x^2 & x \ge 1 \\ 3x + 2 & x < 1 \end{cases}$

<u>QOD</u>: Which of the twelve basic functions are identical except for a horizontal shift?

Syllabus Objectives: 1.2 – The student will solve problems using the algebra of functions. 1.3 – The student will find the composition of two or more functions.

Function Operations

- Sum $(f+g)(x)=f(x)+g(x)$
- Difference $(f-g)(x)=f(x)-g(x)$
- Product $(fg)(x)=f(x)g(x)$
- Quotient $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)},\ g(x)\neq 0$

The domains of the new functions consist of all the numbers that belong to both of the domains of all of the original functions.

Ex1: Find the following for $f(x)=(x+1)^2$ and $g(x)=x-3$.

a) $(f-g)(x)$

$$(f-g)(x)=f(x)-g(x)=(x+1)^2-(x-3)=x^2+2x+1-x+3=\boxed{x^2+x+4}$$

b) $(f/g)(x)$ $\quad (f/g)(x)=\dfrac{f(x)}{g(x)}=\dfrac{(x+1)^2}{x-3}=\boxed{\dfrac{x^2+2x+1}{x-3},\ x\neq 3}$

Note: Domain can also be written as $(-\infty,3)\cup(3,\infty)$.

c) $(f+g)(-2)$

$$(f+g)(-2)=f(-2)+g(-2)=(-2+1)^2+(-2)-3=1-2-3=\boxed{-4}$$

Ex2: Find the rule and domain if $f(x)=\sqrt{9-x^2}$ and $g(x)=\sqrt{x-2}$.

a) $f+g$ $\quad f+g=\boxed{\sqrt{9-x^2}+\sqrt{x-2}}$

Domain of *f*: $9-x^2\geq 0 \Rightarrow 9\geq x^2 \Rightarrow 3\geq |x| \Rightarrow -3\leq x\leq 3$ or $[-3,3]$

Domain of *g*: $x-2\geq 0 \Rightarrow x\geq 2$ or $[2,\infty)$

Domain of $f+g$: $\boxed{[2,3]}$

b) $g+g$ $\quad g+g=\sqrt{x-2}+\sqrt{x-2}=\boxed{2\sqrt{x-2}}$

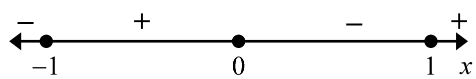
Domain: $x-2\geq 0 \Rightarrow x\geq 2 \Rightarrow \boxed{[2,\infty)}$

Ex3: Find the rule and domain if $f(x)=\sqrt{3x}$ and $g(x)=\sqrt{3x^3-3x}$.

a) fg $\quad fg=\sqrt{3x}\cdot\sqrt{3x^3-3x}=\sqrt{9x^4-9x^2}=\sqrt{9x^2\left(x^2-1\right)}=\boxed{3|x|\sqrt{x^2-1}}$

Domain of *f*: $3x\ge 0 \Rightarrow x\ge 0$ or $[0,\infty)$

Domain of *g*: $3x^3-3x\ge 0 \Rightarrow 3x\left(x^2-1\right)\ge 0$ Critical Points: $x=-1,0,1$

Sign Chart: 

$[-1,0]\cup[1,\infty)$

Domain of *fg*: $\boxed{[1,\infty)}$

b) ff $\quad ff=\sqrt{3x}\cdot\sqrt{3x}=\sqrt{9x^2}=\boxed{3|x|}$ Recall: $\sqrt{x^2}=|x|$

Domain of *ff*: $\boxed{[0,\infty)}$

c) f/g $\quad f/g=\sqrt{\dfrac{3x}{3x\left(x^2-1\right)}}=\boxed{\sqrt{\dfrac{1}{x^2-1}}}$

Domain of f/g: $x^2-1\ne 0$ $\boxed{(1,\infty)}$

Composition of Functions: The **composition** *f* of *g* uses the notations $f\circ g=(f\circ g)(x)=f(g(x))$

- This is read "*f* of *g* of *x*".
- In the composition *f* of *g*, the domain of *f* intersects the range of *g*.
- The domain of the composition functions consists of all *x*-values in the domain of *g* that are also $g(x)$-values in the domain of *f*.

Ex4: Find the rule and domain for $f(x)=\sqrt{x}$ and $g(x)=x^2-5$.

a) $(f\circ g)(x)$ $\quad (f\circ g)(x)=f(g(x))=\sqrt{x^2-5}$

Domain: $x^2-5\ge 0 \Rightarrow \left(-\infty,-\sqrt{5}\right]\cup\left[\sqrt{5},\infty\right)$

b) $g\circ f$ $\quad g\circ f=g(f(x))=\left(\sqrt{x}\right)^2-5=x-5$

Domain: $x\ge 0 \Rightarrow [0,\infty)$

<u>Decomposing Functions</u>

Ex5: Find $f(x)$ and $g(x)$ if $f(g(x)) = (x-1)^2 + 2(x-1) - 7$.

The "inside" function, $g(x)$ is $(x-1)$. So $f(x) = (g(x))^2 + 2(g(x)) - 7$.

Solution: $f(x) = x^2 + 2x - 7$ and $g(x) = x - 1$

<u>Implicitly-Defined Function</u>: independent and dependent variables are on one side of the equation

Implicit: $x + y = 6$ Explicit: $y = -x + 6$

Ex6: Find two functions defined implicitly by the given relation. $3x^2 - y^2 = 15$

Solve for *y*:

$3x^2 - y^2 = 15 \Rightarrow y^2 = 3x^2 - 15$

$y = \sqrt{3x^2 - 15}$ or $y = -\sqrt{3x^2 - 15}$

<u>Relation</u>: set of ordered pairs Note: Functions are special types of relations.

Ex7: Describe the graph of the relation $x^2 + y^2 = 9$.

The graph of this relation is a circle with center $(0,0)$ and radius 3. A circle is not a function because the equation relates two different *y*-values with the same *x*-value. For example, if $x = 0$, $y = 3$ or $y = -3$.

<u>You Try:</u> Find $f(x)$ and $g(x)$ if $(g \circ f)(x) = \sqrt{1 - x^3}$. Is your answer unique?

<u>QOD</u>: Describe how to find the domain of a composite function.

Syllabus Objectives: 1.10 – The student will solve problems using parametric equations. 1.5 – The student will find the inverse of a given function. 1.6 – The student will compare the domain and range of a given function with those of its inverse.

Parametric Equations: a pair of continuous functions that define the *x* and *y* coordinates of points in a plane in terms of a third variable, *t*, called the **parameter.**

Ex1: Find (x, y) determined by the parameters $-2 \le t \le 3$ for the function defined by the equations $x = t - 1,\ y = t^2 + 3$. Find a direct relationship between *x* & *y* and indicate if the relation is a function. Then graph the curve.

Create a table for *t*, *x*, & *y*.

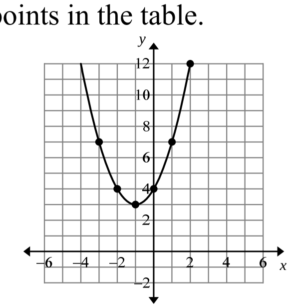
t	$x = t - 1$	$y = t^2 + 3$
-2	-3	7
-1	-2	4
0	-1	3
1	0	4
2	1	7
3	2	12

Solve for *t* and substitute to find a direct relationship between *x* and *y*.

$$x = t - 1 \Rightarrow t = x + 1 \qquad y = t^2 + 3 \Rightarrow y = (x+1)^2 + 3$$

$$\boxed{y = x^2 + 2x + 4}$$

Graph by plotting the points in the table.



Application – Parametric Equations

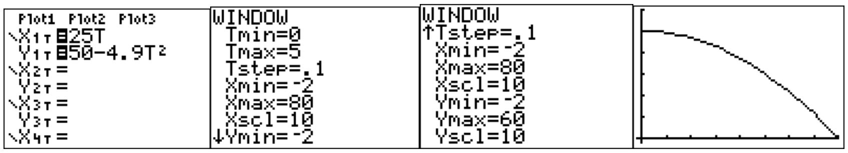
Ex2: A stuntwoman drives a car off a 50 m cliff at 25 m/s. The path of the car is modeled by the equations $x = 25t$ & $y = 50 - 4.9t^2$. How long does it take to hit the ground and how far from the base of the cliff is the impact?

The car hits the ground when $y = 0$. $\qquad 0 = 50 - 4.9t^2 \Rightarrow t = \pm\sqrt{10.204} \Rightarrow \boxed{t \approx 3.194\,\text{sec}}$

The distance from the base of the cliff is *x*. $\qquad x = 25t \Rightarrow x = 25(3.194) \approx \boxed{79.85\,\text{m}}$

Parametric Equations on the Graphing Calculator

Ex: Graph the situation in the previous example on the calculator.
Change to Parametric Mode.
Type each equation into the Y= menu and choose an appropriate window.



Teacher Note: Emphasize to students that this is the path of the car. Have students discuss the best viewing window for the graph.

Inverse: An **inverse relation** contains all points (b,a) for the relation with all points (a,b).

- **Notation**: If $g(x)$ is the inverse of $f(x)$, then $g(x)=f^{-1}(x)$.

  ▽Caution: f^{-1} is NOT *f* to the −1 power. It is the INVERSE of *f*.
- Inverses are **reflections** over the line $y=x$.
- The inverse of a relation will be a **function** if the original relation passes the **Horizontal Line Test** (a horizontal line will not pass through more than one point at a time).
- The **composition** of inverse functions equals the **identity function**, $y=x$.

  So, if $g(x)=f^{-1}(x)$, then $f(g(x))=g(f(x))=x$.

The domain of *f* is the range of f^{-1}. The range of *f* is the domain of f^{-1}.

Ex3: Confirm that *f* and *g* are inverses. $f(x)=x^3-1,\ g(x)=\sqrt[3]{x+1}$

Find the composite functions $f(g(x))$ and $g(f(x))$.

$$f(g(x))=\left(\sqrt[3]{x+1}\right)^3-1=x+1-1=x \quad ☺$$

$$g(f(x))=\sqrt[3]{x^3-1+1}=\sqrt[3]{x^3}=x \quad ☺$$

One-to One Function: a function whose inverse is a function; must pass both the vertical and horizontal line tests

Steps for Finding an Inverse Relation

1) Switch the *x* and *y* in the relation.
2) Solve for *y*.

Ex4: Find the inverse relation and state the domain and range. Verify your answer graphically and algebraically. $f(x)=\dfrac{2x}{x-3}$

The function can be written $y=\dfrac{2x}{x-3}$. Switch *x* and *y*. $x=\dfrac{2y}{y-3}$

$$x(y-3)=2y \Rightarrow xy-3x=2y \Rightarrow xy-2y=3x$$

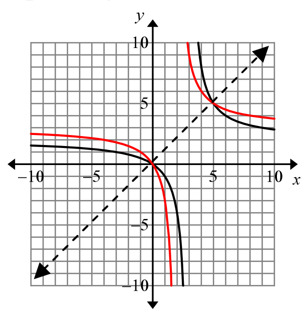
Solve for *y*.

$$y(x-2)=3x \Rightarrow y=\frac{3x}{x-2} \Rightarrow \boxed{f^{-1}(x)=\frac{3x}{x-2}}$$

Domain of *f*: $(-\infty,3)\cup(3,\infty)$ Range of *f*: $(-\infty,2)\cup(2,\infty)$

Domain of f^{-1}: $(-\infty,2)\cup(2,\infty)$ Range of f^{-1}: $(-\infty,3)\cup(3,\infty)$

Verify graphically that this is the inverse (show that f^{-1} is the reflection of *f* over the line $y=x$).



Verify algebraically using composite functions.

$$f\left(f^{-1}(x)\right)=\frac{2\left(\frac{3x}{x-2}\right)}{\frac{3x}{x-2}-3}\cdot\frac{x-2}{x-2}=\frac{6x}{3x-3(x-2)}=\frac{6x}{6}=x \quad ☺$$

$$f^{-1}\left(f(x)\right)=\frac{3\left(\frac{2x}{x-3}\right)}{\frac{2x}{x-3}-2}\cdot\frac{x-3}{x-3}=\frac{6x}{2x-2(x-3)}=\frac{6x}{6}=x \quad ☺$$

<u>You Try</u>: For the parametric equations, find the points determined by integer values of *t*, $-3\le t\le 3$. Find a direct algebraic relationship between *x* and *y*. Then graph the relationship in the *xy*-plane.

$$x=t-3,\ y=\sqrt{t}$$

<u>QOD</u>: Explain why the domain and range are "switched" in the inverse of a function.

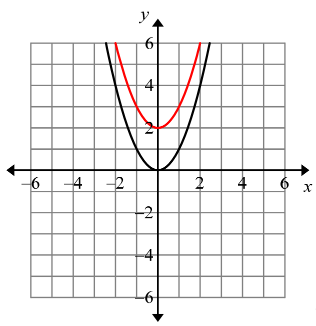
Syllabus Objective: 2.8 – The student will construct the graph of a function under a given translation, dilation, or reflection.

Rigid Transformation: leaves the size and shape of a graph unchanged, such as translations and reflections

Non-Rigid Transformation: distorts the shape of a graph, such as horizontal and vertical stretches and shrinks

Vertical Translation: a shift of the graph up or down on the coordinate plane

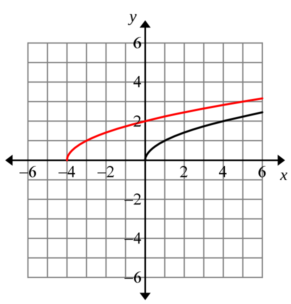
Ex1: Graph $y_1 = x^2$ and $y_2 = x^2 + 2$ on the same coordinate plane. Describe the transformation.



Vertical Shift UP 2 units.

Vertical Translation: $y = f(x) +/- c$ is a translation up/down c units.

Ex2: Graph $y_1 = \sqrt{x}$ and $y_2 = \sqrt{x+4}$ on the same coordinate plane. Describe the transformation.



Horizontal Shift LEFT 4 units.

Horizontal Translation: $y = f(x +/- c)$ is a translation left/right c units.

Reflection: a flip of a graph over a line

- $y = -f(x)$ is a reflection of $y = f(x)$ across the x-axis.
- $y = f(-x)$ is a reflection of $y = f(x)$ across the y-axis.

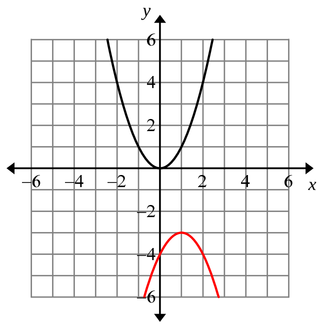
Teacher Note: Have students plot points to see this relationship.

Ex3: Describe the transformations of $y=-(x-1)^2-3$ on $y=x^2$. Then sketch the graph.

$y=-(x-1)^2-3$ Reflection across the *x*-axis.

$y=-(x-1)^2-3$ Translate RIGHT 1 unit.

$y=-(x-1)^2-3$ Translate DOWN 3 units.



Horizontal Stretches or Shrinks

$y=f\left(\frac{x}{c}\right)$ is a horizontal **stretch** by a factor of *c* of $f(x)$ if $c>1$

$y=f\left(\frac{x}{c}\right)$ is a horizontal **shrink** by a factor of *c* of $f(x)$ if $c<1$

Vertical Stretches or Shrinks

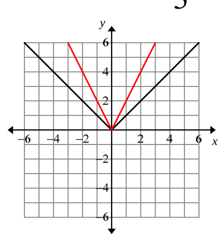
$y=c\cdot f(x)$ is a vertical **stretch** by a factor of *c* of $f(x)$ if $c>1$

$y=c\cdot f(x)$ is a vertical **shrink** by a factor of *c* of $f(x)$ if $c<1$

Ex4: Graph the following on the same coordinate plane and describe the transformation of y_1.

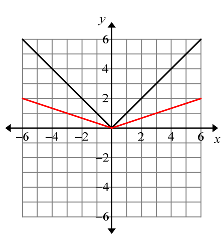
$y_1=|x|,\ y_2=2|x|,\ y_3=\frac{1}{3}|x|$

$y_1=|x|,\ y_2=2|x|$



y_2 is a vertical stretch of y_1 by a factor of 2.

$y_1=|x|,\ y_3=\frac{1}{3}|x|$

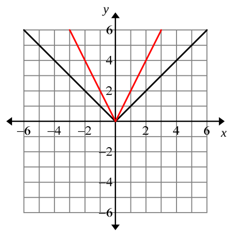


y_3 is a vertical shrink of y_1 by a factor of $\frac{1}{3}$.

Ex5: Graph the following on the same coordinate plane and describe the transformation of y_1.

$$y_1 = |x|,\ y_2 = |2x|,\ y_3 = \left|\frac{x}{3}\right|$$

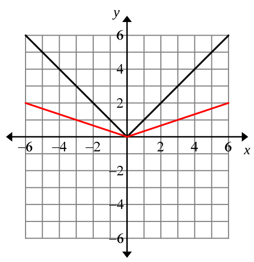
$y_1 = |x|,\ y_2 = |2x|$



y_2 is a horizontal shrink of y_1 by a factor of $\frac{1}{2}$.

Note: $y_2 = |2x| = \left|\frac{x}{\frac{1}{2}}\right|$

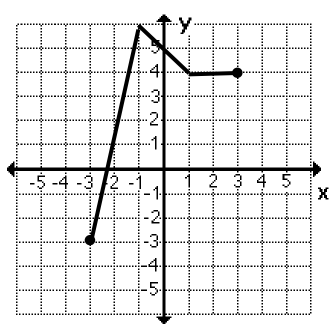
$y_1 = |x|,\ y_3 = \left|\frac{x}{3}\right|$

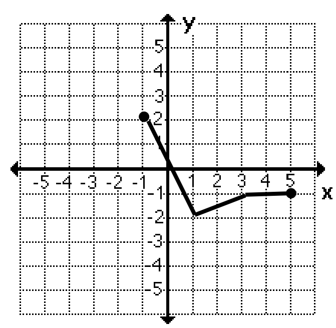


y_3 is a horizontal stretch of y_1 by a factor of 3.

Note: A horizontal stretch causes a vertical shrink and a horizontal shrink causes a vertical stretch.

Ex: The function shown in the graph is $f(x)$. Sketch the graph of $y = -\frac{1}{2} f(x-2) + 1$.





Solution:

- Vertical Shrink by a factor of ½.
- Reflect over the *x*-axis.
- Shift right 2 units and up 1 unit.

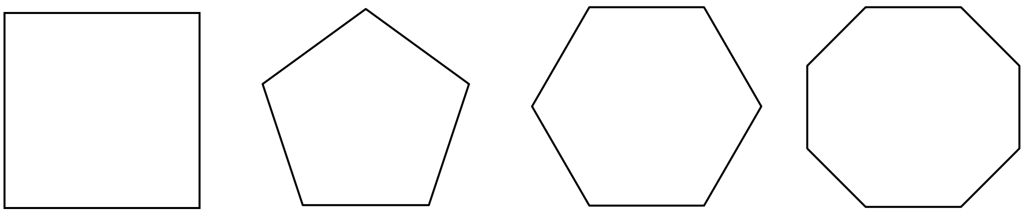
<u>You Try:</u> Use the graph of $y = \sqrt{x}$. Write the equation of the graph that results after the following transformations. Then apply the transformations in the opposite order and find the equation of the graph that results. Shift left 2 units, reflect over the *x*-axis, shift up 4 units.

<u>QOD</u>: Does a vertical stretch or shrink affect the graph's *x*-intercepts? Explain why or why not.

Syllabus Objective: 1.11 – The student will set up functions to model real-world problems.

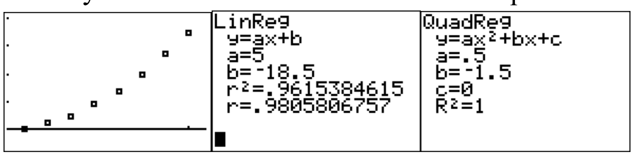
<u>Modeling a Function Using Data</u>

Exploration Activity:
Create a table that relates the number of diagonals to the number of sides of a polygon. Use the graphing calculator to graph a scatter plot and find at least two regression equations and their r and r^2 values.



n	d
3	0
4	2
5	5
6	9
7	14
8	20
9	27
10	35

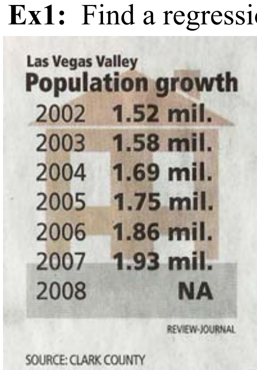
Note by the second differences that this is quadratic!



Teacher Note: If a student does not see r and r^2 values on their calculator, use the command DiagnosticsOn in the Catalog. Also – After the command QuadReg, type in Y1 (in the VARS menu) to have the function automatically appear in Y1 to graph.

<u>Correlation Coefficient (r) & Coefficient of Determination (r^2):</u> The closer the absolute value is to 1, the better the curve fits the data.

Ex1: Find a regression equation and its r and r^2 values. Then predict the population in 2020.

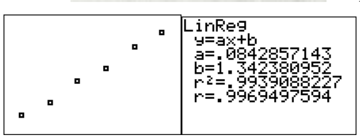


Las Vegas Valley
Population growth

2002	1.52 mil.
2003	1.58 mil.
2004	1.69 mil.
2005	1.75 mil.
2006	1.86 mil.
2007	1.93 mil.
2008	NA

REVIEW-JOURNAL
SOURCE: CLARK COUNTY

Let $x = 0$ be the year 2000 and make a scatter plot.



$\boxed{y = 0.084x + 1.342}$ $\quad y(20) = 0.084(20) + 1.342 = \boxed{3.02\text{ million}}$

Modeling Functions with Equations

Ex2: A beaker contains 400 mL of a 15% benzene solution. How much 40% benzene solution must be added to produce a 35% benzene solution?

Write the equation that models the problem: $0.15(400)+0.4x=0.35(400+x)$

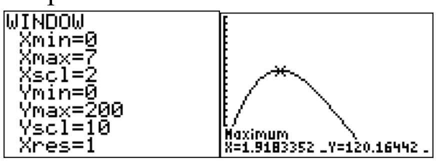
Solve the equation: $60+0.4x=140+0.35x \Rightarrow 0.05x=80 \Rightarrow x=\boxed{1600\,\text{mL}}$

Ex3: An open box is formed by cutting squares from the corners of a 10 ft by 14 ft rectangular piece of cardboard and folding up the flaps. What is the maximum volume of the box?

Let *x* be the length of the sides of the squares cut from the corners.

Write the volume *V* of the box as a function of *x*. $V=lwh=(14-2x)(10-2x)x$

Graph *V* on the calculator and find the maximum value.



The maximum volume is approximately $\boxed{120\text{ ft}^3}$.

You Try: How many rotations per second does a 24 in. diameter tire make on a car going 35 miles per hour?

QOD: Describe how to determine which regression equation to use on the graphing calculator to represent a set of data.