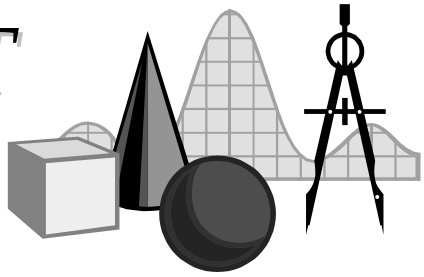
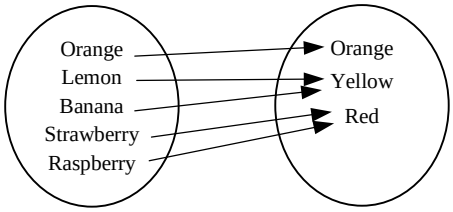
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

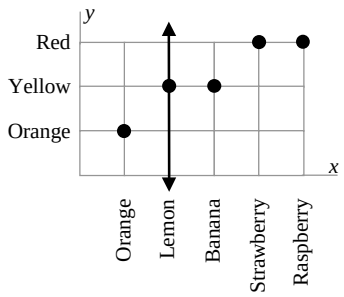
Math Audit Team
Regional Professional Development Program
December 4, 2000 — High School Edition





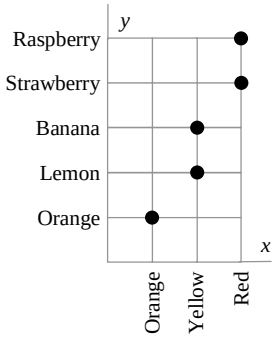
This issue of *Take It to the MAT* is fourth and last in a series on relations and functions. We will address graphical utilities to determine if a relation is a function and whether a function has an inverse. Namely, these are the *vertical line test* and the *horizontal line test*.

Let's go back to the definition of a function: *A function is a relation in which each element of a set X, is paired with a unique element of set Y.* And let's look at a previous example where X is a set of fruits and Y is a set of colors where the relation is pairing each fruit with its color. Rather than examine the relation as shown above, we can also look at a graph of it on a coordinate plane. Let $x \in X$ and $y \in Y$.

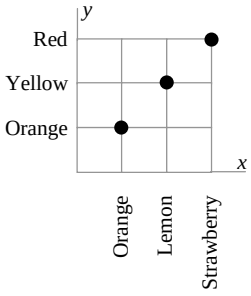


For each element in X, it is paired with a unique element in Y. If x = Lemon, there is only one value of y associated with it, Yellow. The same is true for the other four fruits. Think about the statement x = Lemon; it is the equation of a vertical line. When we combine that fact with the definition of a function, we create the *vertical line test: If every vertical line drawn on the graph of a relation passes through no more than one point on the graph, then the relation is a function.* From this, we can see that our fruity relation is a function.

If we look at the inverse relation—matching the color with the fruit—we can see it is not a function by the vertical line test. We know from the last issue of *Take It to the MAT* that a function must be one-to-one to have an inverse that is a function. That is, each member of X is paired with exactly one member from Y, and vice versa.



Considering that in a one-to-one function each member from Y is matched with only one X, and that a given value of y can be thought of as a horizontal line, that line may pass through only x. In other words, for a function to have an inverse that is also a function, it must pass the *horizontal line test*: *If every horizontal line drawn on the graph of a function passes through no more than one point on the graph, then the function is one-to-one and has an inverse that is a function.* This is shown in the last diagram.



The vertical line and horizontal line tests are based on the rules and definitions of functions and inverse functions. We should not blindly throw utilities like the two line tests at students without the conceptual background as to what they mean and why they work.