



TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



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“You can’t take the square root of a negative number.”

How many times have we heard that statement in a pre-algebra or algebra class? More times than we can count, very likely. But is it true? If it is, why is it true? If not, who told us a fib? In this issue of *Take It to the MAT*, we’ll address those questions.

First, let’s start with the definition of square root.

The square root of a number n is a number that when multiplied by itself equals n .

Sounds pretty simple. The square root of 4 is 2, because $2 \times 2 = 4$. The square root of 25 is 5, because $5^2 = 25$. The square root of 0 is 0, because $0^2 = 0$. No problem.

Here comes a little twist, though. Isn’t the square root of 4 also -2 , because $(-2) \times (-2) = 4$? Isn’t the square root of 25 also -5 , because $(-5)^2 = 25$? The short answers to those questions are both, “Yes.”

Any number has two square roots because there are two numbers, one positive and one negative that when multiplied by each other give the original number. Both $+1$ and -1 are square roots of 1, because $(+1)^2 = (-1)^2 = 1$. (Zero is a bit quirky. Technically, it also has two square roots, but they’re both zero.)

For numbers other than perfect squares, the same is true. The number 3 has two square roots:

$\sqrt{3}$ and $-\sqrt{3}$. This is because $\sqrt{3} \times \sqrt{3} = 3$ and $(-\sqrt{3}) \times (-\sqrt{3}) = 3$. We could also express this as $(\sqrt{3})^2 = (-\sqrt{3})^2 = 3$.

What about negative numbers—what is the square root of -4 ? What number multiplied by itself is -4 ? It’s not 2, because $2^2 = 4$. It’s not -2 , because $(-2)^2 = 4$. It turns out that in the *real number system*, which is what we restrict ourselves to in first-year algebra and the courses before it, negative numbers don’t have square roots. So, as long as we live only in the world of real numbers, “You can’t take the square root of a negative number.” This is because there is no real number that when squared equals a negative number. (If we allow ourselves to enter the world of *imaginary* numbers, then square roots of negative numbers are allowed. For more on that, see *Take It to the MAT*, High School Edition, January 31, 2000, entitled *Product of Radicals Rules*.)

Finally, we need to take a quick step back and look at the radical symbol $\sqrt{\quad}$. We learned above that the number n has two square roots. When we see the mathematical expression \sqrt{n} , we are to report only the *principal* square root, that is, the positive value. So, while 16 has two square roots, 4 and -4 , the expression $\sqrt{16}$ equals $+4$. When that radical shows up in the context of a square root, the answer can **only** be positive (or zero in the case of $\sqrt{0}$). For this reason, one often sees the following equation in textbooks: $\sqrt{x^2} = |x|$. Since we don’t know if x is positive or negative (or zero), the way we guarantee the square root is positive is to use the absolute value of x .