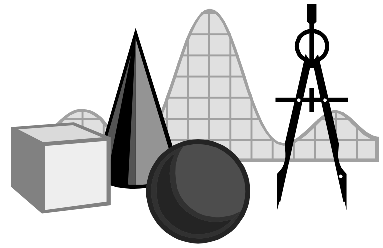
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Regional Professional Development Program
November 5, 2001 — Middle School Edition

One of the big ideas in algebra, actually in all of mathematics, is the concept of *functions*. This *Take It to the MAT* will address three questions about functions: What is a *function*? What is *domain*? And what is *range*?

A *function* shows the relationship between two sets of objects. Typically, those objects are numbers, but they don't have to be. Generally, some rule or property defines the relationship between the sets. For example, let's suppose my first set of objects is all of the items on the menu at a restaurant and the second set of objects is all of the prices listed on the menu. We can define a function that matches each item with its price. The function's rule would simply be, "Match the menu item with its price."

The *domain* of a function is all of those objects in the first set. In the example, it is the set of menu items. When evaluating functions, we take members of the domain—the first set—and apply the function rule to them. The result of the rule's application is a second set of objects called the *range*. In our restaurant example, the range is the set of prices.

The obvious conclusion is that all relationships between sets of objects are functions. Nothing could be further from the truth. One tidbit of information has been left out up to this point. What makes functions special is that each element in the domain is paired with one and only one element in the range. That is, any member of the first set can only have one match when the function rule is applied. In the restaurant example, this is satisfied because each item on the menu (member of the domain) has only one price (member of the range).

This "uniqueness" property is the key to functions. It only has to work in one direction, however, from domain to range. The opposite is not necessarily true. Going back to our menu, let's look at the cheeseburger and the chef's salad. The cheeseburger is $4.95 and the chef's salad is $5.25. No problem, each has only one price. But, upon further examination, the club sandwich is also $5.25. Problem? No problem! Each member in the domain (of items) has only one member in the range (of prices) associated with it. That's the definition of a function. It does not concern us that more than one domain element has the same range element.

Here's an example using numbers for the sets and an equation as the rule. Let the domain be the set of integers {–2, 0, 2, 4}. The function rule pairs each member of the domain with its square. Notationally, we would write this $f(x) = x^2$; that is, "the value of the function (named '*f*') at *x* is *x*-squared." The range of the function is the set of numbers when the rule is applied to the domain, namely $\{(-2)^2, 0^2, 2^2, 4^2\}$ or {4, 0, 4, 16}. We don't write repeated values in a set, so the range is actually {0, 4, 16}. Notice that both –2 and 2 match up with 4. That's OK, because –2 is paired with only one value and 2 is paired with only one value. The uniqueness only need work in the direction from domain to range.

An example of where a relationship is *not* a function would be as follows: Each element of the domain a is paired with elements in the range b, such that $4a - b^2 = 0$, where a comes from the set {0, 1}. If a is 0, then b is also 0—no problem there. But, if $a = 1$, then $b = 2$ or $b = -2$. There are two values of b that pair up with a single value of a. This relationship is *not* a function.

A quick, non-numerical example would be letting the domain be all students in a class. Our relationship rule will match each student with the chair in which he is sitting. If two students shared a chair, that's alright because each is sitting in a single chair. But, a single student would not be allowed to sit in two chairs simultaneously and have the relationship still be a function, because a member of the domain is not paired with a unique element in the range.