Vocabulary: Define each word and give an example.

1. augmented matrix

2. row echelon form

3. feasible region

Short Answer:

4. When solving a linear programming program, what points are necessary to test in the objective function to maximize or minimize?

5. Do all square matrices have inverses? Explain why or why not.

6. Describe the graph of a system of two linear equations with no solution.

Review:

7. Given that $P = (4, -1)$ and $Q = (7, -2)$, find the component form and magnitude of the vector $\overrightarrow{PQ}$.

8. Evaluate exactly:
   a. $\csc \frac{2\pi}{3}$
   b. $\tan \frac{3\pi}{4}$
   c. $\cos 270^\circ$
9. Find the inverse of the matrix. Then use matrix multiplication to verify your result.
\[
\begin{bmatrix}
2 & 3 \\
6 & 4
\end{bmatrix}
\]

10. Write the system of equations for the augmented matrix. Do not solve.
\[
\begin{bmatrix}
2 & 1 & 0 & 3 \\
-1 & 3 & 4 & 0 \\
0 & -2 & 1 & 5
\end{bmatrix}
\]

11. Solve the system of equations using Gaussian elimination.
\[
\begin{align*}
2x + 3y - z &= 4 \\
2x - 3y + z &= 14 \\
4x + y - 2z &= 7
\end{align*}
\]

\[
\begin{align*}
2x + 3y - 12z &= 1 \\
x - 2y + z &= 4 \\
4x + y - 14z &= 7
\end{align*}
\]
13. Determine which elementary row operation(s) applied to the first matrix will yield the second matrix.

\[
\begin{bmatrix}
3 & 6 & -2 & 5 \\
4 & 7 & 5 & -2 \\
-3 & 2 & 0 & 3
\end{bmatrix}, \quad \begin{bmatrix}
15 & 27 & 13 & -1 \\
4 & 7 & 5 & -2 \\
-3 & 2 & 0 & 3
\end{bmatrix}
\]

A. 5R₁  B. 3R₂ + R₁  C. 3R₁ + R₂  D. 3R₂ - R₃

14. Find a **reduced row echelon form** for the matrix.

\[
\begin{bmatrix}
1 & 2 & -3 & 3 \\
4 & 1 & 2 & -2 \\
2 & -3 & 8 & 5
\end{bmatrix}
\]

15. Find the partial fraction decomposition. \[
\frac{2x}{x^2 + 2x - 3}
\]

16. Find the partial fraction decomposition. \[
\frac{x^3 - 2}{x^2 + x}
\]
17. Graph the inequality $x^2 + y^2 \leq 9$

18. Write an inequality whose solution set matches the graph.

19. Graph the system of inequalities.

\[(x - 2)^2 + (y + 1)^2 \leq 16\]
\[y \geq -\frac{1}{2}x + 1\]

20. Write a system of inequalities whose solution set is the region shown.
21. Pump A can fill a tank in 8 hours. Pump B can fill the tank in 6 hours. How long will it take to fill the tank using both pumps?

22. Trail Snax Corp. mixes raisins that cost $5.00 per kg with peanuts that cost $3.80 a kg. How many kilograms of raisins should be mixed with 10 kg of peanuts to obtain a mixture worth $4.00 per kg?

23. Merlin has $1600 more invested at 5% than she does at 8%. The annual return from the 5% investment is $17 more than the annual return from the 8% investment. How much is invested at each rate?
Name: _______________________________ Period: ___ Date: __________

CALCULATOR SECTION

24. Solve the system of equations graphically.
\[ y = \frac{1}{x} \]
\[ y = 3x^3 - x^6 \]
\[ x - 2y + z - w = 2 \]
\[ 2x + y - z - w = -1 \]

25. Solve the system of equations.
\[ x - y + 2z - w = -1 \]
\[ x + 3y - z + w = 4 \]

26. Determine the function so that its graph \( f(x) = ax^3 + bx^2 + cx + d \) contains the points \((2, 8), (4, 5), (6, 3), \) and \((9, 4)\).

27. Find the partial fraction decomposition of \( \frac{2x^3 - x^2 + 5x}{(x^2 + 1)^2} \)
28. Find the maximum values of the objective function, \( f = 3x + 5y \) subject to the following constraints:
\[
\begin{align*}
3x + 2y & \geq 20 \\
5x + 6y & \geq 52 \\
2x + 7y & \geq 30 \\
x, y & \geq 0
\end{align*}
\]

29. Determine the number of solutions to the system of equations represented by the augmented matrix.
\[
\begin{bmatrix}
1 & 0 & 0 & 4 & 3 \\
0 & 1 & 0 & 3 & -1 \\
0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]