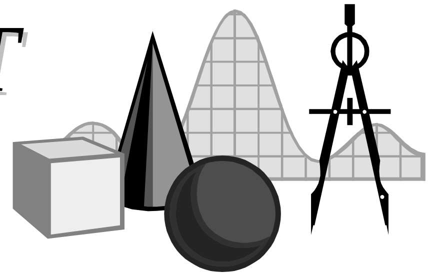
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
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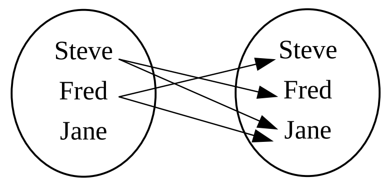
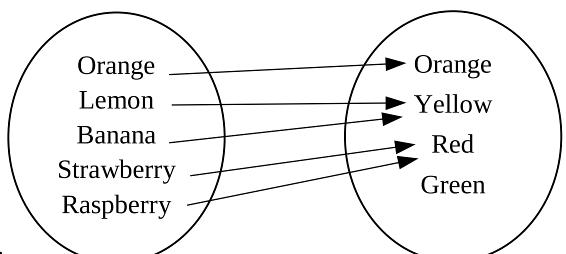
In the last issue of *Take It to the MAT*, we took a good look at relations. To review, a relation is *an association between, or property of, two or more objects*. In this edition we will examine functions, a special type of relation.

First, the definition of a function: *A function is a relation in which each element of a set X , called the domain, is paired with a unique element of set Y, called the codomain. The set of elements in Y that are paired with an element in X is called the range.* (The range is a subset of the codomain.) The key word in the definition is *unique,* that is for every element in the domain there is *one and only one* element in the range.



Let's look at an example from the last issue. A family consists of the children Steve, Fred, and Jane. We will choose the relation, "is a brother of" to pair the set {Steve, Fred, Jane} with itself. The relation would be written as {(Steve, Fred), (Fred, Steve), (Steve, Jane), (Fred, Jane)}. Is this a function? The answer is absolutely not. As the mapping diagram at right shows, some elements in the first set are paired with more than one (or none) of the elements in the second set. To be a function, *each* element in the first set must pair with *exactly one* in the second set. The *domain* of the relation is the set of all of first elements of the ordered pairs, namely {Steve, Fred). The *codomain* is the entire second set, namely {Steve, Fred, Jane}. The *range* is the set of all second elements of the ordered pairs and is this case is also {Steve, Fred, Jane}.



How about this one? First Set = {Orange, Lemon, Strawberry, Banana, Raspberry}; Second Set = {Red, Orange, Yellow, Green}. Examine the relation that matches each fruit with its general color. Each element of the first set is paired with one and only one element in the second set, so this relation is a function.

In the first example, the fact that Steve and Jane both pair with two elements in the codomain, *and* that Jane does not pair at all, violate the definition of function. In the second example, it's not a problem that Green in the codomain is not paired with anything, nor does it matter that Red and Yellow are each paired with two elements in the domain. Since *each* member of the domain is paired with *exactly one* element in the codomain, the relation *is* a function.

A note about *range*. The range of the fruit example is {Orange, Yellow, Red}; it does not include Green because Green is not paired with any element in the domain. We must remember that the range is what values the domain is *actually* paired with, not potentially paired with. Again, the range is a subset of the codomain.

The term codomain is not one that is important for high school students to know. It is presented here for the reader's edification. Domain and range are the key terms that should be studied.

Next time: one-to-one functions, multi-valued functions, and inverse functions.