TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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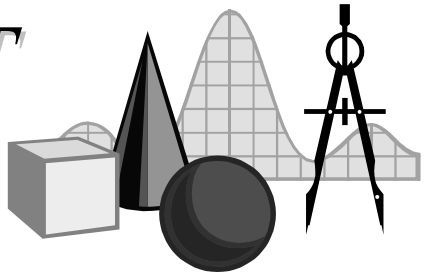
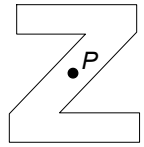
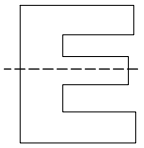


In this issue of *Take It to the MAT* we will look at symmetry. There are misconceptions and misusages of language that occur when studying symmetry and we will clear them up in the paragraphs below.

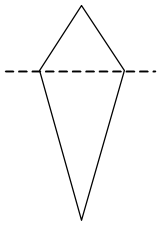
There are three basic types of symmetry: point, line, and plane. Since all things exist in three dimensions, everything potentially has symmetry with respect to a plane. The focuses of this paper are point and line symmetry; plane symmetry will not be addressed further.

A figure that can be rotated less than 360° onto itself has point symmetry. The rotation must be made about a point called the *point of symmetry*. The "Z" shape has point symmetry because it can be rotated 180° onto itself about the point *P*. We say that the shape "has symmetry about the point *P*" or "is symmetric with respect to the point *P*." Simply stating, "It is symmetric," is not enough. One must include to what or about what the figure is symmetric. The "E" shape does not have point symmetry.



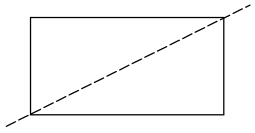


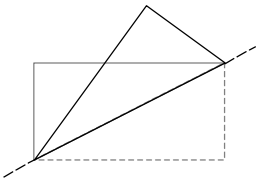
Now on to the main topic, line symmetry. A figure has line symmetry if one can fold the figure so that it has two parts that match exactly. That is, the two "halves" lie on each other perfectly and completely. If this is the case, the fold is called the *line of symmetry*. We then say that the figure "has symmetry about the line" or "is symmetric with respect to the line." Again, it is not correct to say simply, "It is symmetric." The "E" shape is symmetric with respect to the line shown. The kite shape does not have symmetry about the line shown.



Since the "halves" of shapes with line symmetry lie on each other perfectly and completely, they are *congruent*. Congruent figures have the property that they have the same area. Thus, the two shapes on either side of the line of symmetry are indeed *halves*. If we can draw a line of symmetry through a plane figure, then the two sides are congruent and have equal areas.

A mistake that is often made is inferring the converse of the last sentence in the paragraph above. That is, if two parts of a figure separated by a line are congruent and have equal areas, then the line is a line of symmetry. This is not the case as can be seen in the rectangle at right. The two triangles are congruent and both have equal areas—half of the rectangle—but the diagonal is not a line of symmetry. If the figure were folded on the diagonal, the two sides would not match perfectly and completely.





Don't forget that a figure may have more than one line of symmetry. How many lines of symmetry do each of the figures on this page have? How many lines of symmetry does a circle have?

Final thought: Can a figure have more than one point of symmetry?