Review Vocab.: Complementary, Supplementary and Vertical angles.

Syllabus Objective: 2.1 - The student will differentiate among definitions, postulates, corollaries, and theorems.

Definition - a statement defining an independent/dependent relationship between two conditions, the hypothesis and conclusion. Often written in the 'if-then' form. Can be written as a true biconditional.

Postulate (axiom) - is a statement (basic assumption) assumed to be true without proof.

Theorem - is a statement that has to be proved.

Corollary - is a special case of a theorem. Follows from information proven in the theorem.

Syllabus Objective: 2.2 - The student will justify conjectures and solve problem using inductive reasoning.

Conditional Statement - A type of logical statement that has two parts, a hypothesis and a conclusion. Conditional statements are often written in 'if-then' format.

Inductive Reasoning - Reasoning that uses specific examples to arrive at a logical conclusion. Conclusions are not logical certainty, but the best 'guess' based on what has been observed. Given a pattern of numbers and assuming that it continues is an application of inductive reasoning.

Ex) The postman delivers mail Monday through Thursday at 1:00 PM. We could conclude that he will deliver mail at 1:00 PM on Friday.

?: What happens if he has a flat tire? Or an accident?

Ex) It has been hot and sunny for 5 days straight. We might conclude that tomorrow will be hot and sunny.

?: Could there be a freak cold snap?
Ex) Find the next item in the sequence:
Movie show times: 8:30 AM, 9:45 AM, 11:00 AM, 12:15 PM, ... 1:30 PM

? : How far apart are the movies show times spaced? 1 hour and 15 minutes.

Syllabus Objective: 2.3 – The student will differentiate between deductive and inductive reasoning.

Deductive Reasoning – Reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

Symbolic notation of conditional statements:

- $p$: 'hypothesis condition'
- $q$: 'conclusion condition'
- $\rightarrow$: 'implies'
- $\leftrightarrow$: 'if and only if'
- $\sim$: 'not' or negation

If $p$, then $q$ ... $p \rightarrow q$ ... $p$ implies $q$
If not $q$, then not $p$ ... $\sim q \rightarrow \sim p$
p if and only if $q$ ... $p \leftrightarrow q$

Ex) It is stated that two angles are congruent, from the definition of congruent we can assert that their measures are equal. (This is the type of reasoning used in two-column proofs.)

Law of Detachment: If $p \rightarrow q$ is true and $p$ is true, then $q$ is true.

- Conditional statements do NOT imply that any individual condition exists or is true, only that there is a relationship between the two conditions.
- Detachment applies to a singular conditional statement and the existence (or truth) of the hypothesis condition

If $p \rightarrow q$ is true and $p$ exists, then $q$ exists.
If an animal is a duck, then it is a bird.
My pet is a duck.
Therefore, my pet is a bird.
Ex) If you tease your little brother/sister, then your parents will be upset with you.

You start teasing your sibling, what will the outcome be?

→ Your parents will be upset with you.

?: If your parents are upset with you, does that mean you were teasing your little brother/sister?

→ No, there are probably several reasons why your parents could be upset with you.

**Law of Syllogism:** If the conclusion condition of statement A is the hypothesis condition of statement B, then A and B can be 'linked' together to form a true conditional statement with the hypothesis condition of A and conclusion condition of B.

- Syllogism is a ‘transitive’ process linking two conditional statements in order to form a third conditional statement.
- Like conditional statements, Syllogism does NOT imply that any condition exists or is true.
- Syllogism applies to two conditional statements and allows us to create a third conditional statement by linking the first two.

<table>
<thead>
<tr>
<th>If $p \rightarrow q$ and $q \rightarrow r$, then $p \rightarrow r$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>If an animal is a duck, then it is a bird.</td>
</tr>
<tr>
<td>If an animal is a bird, then it has feathers.</td>
</tr>
<tr>
<td>∴</td>
</tr>
<tr>
<td>If an animal is a duck, then it has feathers.</td>
</tr>
</tbody>
</table>

Ex) If you do your chores, then you can go out with friends this weekend.

If you go out with your friends, then you will go to the movies.

Therefore...

If you do your chores, then you will go to the movies.
Syllabus Objective: 2.4 – The student will distinguish between the hypothesis and conclusion of an implication.

Hypothesis: the independent condition.
Conclusion: the condition dependent upon the hypothesis.

Ex) If an animal is a duck, then it is a bird.
   Hypothesis: an animal is a duck.
   Conclusion: an animal is a bird.

The conclusion is dependent upon the hypothesis. The hypothesis is independent. This is analogous to the x-y relationship in functions.

Syllabus Objective: 2.5 – The student will write an implication as an if-then statement.

Ex) The sum of the measures of two supplementary angles is 180°.
   → If the sum of the measures of two angles is 180°, then they are supplementary.

Ex) Two angles are complementary if the sum of their measures is 90°.
   → If the sum of the measures of two angles is 90°, then they are complimentary.

*Notice the placement of the word ‘if’ in the second example changes the customary placement of ‘if-then’ in the sentence.

Syllabus Objective: 2.6 – The student will analyze conditional or bi-conditional statements.

Biconditional Statement - a conditional statement that is logically equivalent to its converse. A biconditional is a true conditional statement whose converse is also true.

- Both statements can be summarized into one statement using the phrase ‘if and only if’.
- Geometric definitions are biconditional statements.

Ex) Perpendicular lines: Two lines are perpendicular if and only if they intersect to form a right angle.
   → If two lines are perpendicular, then they intersect to form a right angle.
   If two lines intersect to form a right angle, then they are perpendicular lines.
   Two lines are perpendicular if and only if they intersect to form a right angle.
Syllabus Objective: 2.7 – The student will write and analyze converse, inverse, and contra-positive of a statement.

**Converse** - a form of a conditional statement created by ‘switching’ the hypothesis and conclusion.

**Inverse** - a form of a conditional statement created by ‘negating’ both the hypothesis and conclusion.

**Contrapositive** - a form of a conditional statement created by switching AND negating the hypothesis and conclusion.

Ex) *Conditional Statement: If an animal is a duck, then it is a bird.* (True)

**Converse:** If an animal is a bird, then it is a duck. (False)

**Inverse:** If an animal is NOT a duck, then it is NOT a bird. (False)

**Contrapositive:** If an animal is NOT a bird, then it is NOT a duck. (True)

Logically equivalent statements - two conditional statements which are necessarily both true (or both false.) A conditional statement and its contrapositive are logically equivalent. As is the inverse & converse.

Syllabus Objective: 2.8 – The student will find counterexamples to disprove mathematical statements.

**Counterexample** - An example used to show that a given statement is not always true.

Ex) If \( n \) is a real number, then \( -n \) is negative.

**Counterexample:** If \( n = -5 \) (a real number), then \( -n = 5 \) (a positive number).

Ex) If \( n \) is a prime number, then \( n + 1 \) is not prime.

**Counterexample:** If \( n = 2 \) (a prime number), then \( n + 1 = 3 \) (also prime).

Ex) If the area of a rectangle is 20 ft\(^2\), then its length is 10 feet and its width is 2 feet.

**Counterexample:** The length could be 5 feet and the width could be 4 feet and the area would still be 20 ft\(^2\).
Syllabus Objective: 2.9 - The student will write algebraic proofs.

Reasoning with Algebraic Properties

- **Addition Property of Equality**: If \( a = b \), then \( a + c = b + c \).
- **Subtraction Property of Equality**: If \( a = b \), then \( a - c = b - c \).
- **Multiplication Property of Equality**: If \( a = b \), then \( ac = bc \).
- **Division Property of Equality**: If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \).
- **Reflexive Property of Equality**: If \( a \) is a real number, then \( a = a \).
- **Symmetric Property of Equality**: If \( a = b \), then \( b = a \).
- **Transitive Property of Equality**: If \( a = b \) and \( b = c \), then \( a = c \).
- **Substitution Property of Equality**: If \( a = b \), then \( a \) can be substituted in for \( b \) in any expression.

It is often advantageous to show the Addition Property of Equality as follows:

If \( a = b \) and \( c = d \), then \( a + c = b + d \).

This is more typical of how the property will be used in the context of writing a proof.

It is worth discussing how the Subtraction and Division Properties are special cases of the Addition and Multiplication properties where \( c \) is either negative or a reciprocal value, respectively.
Ex) Write an algebraic proof:

Given: \( \frac{5x - 1}{8} = 3 \)

Prove: \( x = 5 \)

<table>
<thead>
<tr>
<th>Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \frac{5x - 1}{8} = 3 )</td>
</tr>
<tr>
<td>2) ( 5x - 1 = 24 )</td>
</tr>
<tr>
<td>3) ( 5x = 25 )</td>
</tr>
<tr>
<td>4) ( x = 5 )</td>
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</tbody>
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<tr>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Given</td>
</tr>
<tr>
<td>2) Multiplication Property of Equality</td>
</tr>
<tr>
<td>3) Addition Property of Equality</td>
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<tr>
<td>4) Division Property of Equality</td>
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Syllabus Objective: 2.10 - The student will write a formal deductive proof.

When writing geometric proofs students should be required to follow certain minimum guidelines. The 'T-chart' should be drawn and labeled. In the upper left quadrant, the question should be stated, including the given and prove statements. In the upper right quadrant, a diagram should be sketched. The diagram should include all given information marked on the sketch. The lower left quadrant should have numbered statements and the lower right should have corresponding numbered reasons that support each statement on the left.

Ex) Write a two-column proof:

Prove that if \( DB \) bisects \( \angle ADC \), then \( \angle 2 \cong \angle 3 \).
Given: \( \overline{DB} \) bisects \( \angle ADC \)

Prove: \( \angle 2 \cong \angle 3 \)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
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<tbody>
<tr>
<td>1) ( \overline{DB} ) bisects ( \angle ADC )</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) ( \angle 1 \cong \angle 2 )</td>
<td>2) Definition of bisect</td>
</tr>
<tr>
<td>3) ( \angle 1 ) and ( \angle 3 ) are vertical angles</td>
<td>3) Definition of vertical angles</td>
</tr>
<tr>
<td>4) ( \angle 3 \cong \angle 1 )</td>
<td>4) Vert. ( \angle s ) are ( \cong )</td>
</tr>
<tr>
<td>5) ( \angle 3 \cong \angle 2 )</td>
<td>5) Transitive Prop. of Cong.</td>
</tr>
<tr>
<td>6) ( \angle 2 \cong \angle 3 )</td>
<td>6) Symmetric Prop. of Cong.</td>
</tr>
</tbody>
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Syllabus Objective: 2.11 - The student will write proofs related to segments and angles.

* Students should write formal, two-column proofs using the following postulates, properties and/or theorems in conjunction with definitions.

Postulates:
- Through any two points there is exactly one line.
- A line contains at least two points.
- If two lines intersect, then their intersection is one point.
- Through any three noncollinear points there exists exactly one plane.
- A plane contains at least three noncollinear points.
- If two points lie in a plane, then the line that contains the points lies in the plane.
- If two planes intersect, then their intersection is a line.
Proving Statements About Segments:
  o Reflexive Property of Segment Congruence: If \( \overline{AB} \) is a segment, then \( \overline{AB} \cong \overline{AB} \).
  o Symmetric Property of Segment Congruence: If \( \overline{AB} \cong \overline{CD} \), then \( \overline{CD} \cong \overline{AB} \).
  o Transitive Property of Segment Congruence: \( \overline{AB} \cong \overline{CD} \) and \( \overline{CD} \cong \overline{EF} \), then \( \overline{AB} \cong \overline{EF} \).

Proving Statements About Angles:
  o Reflexive Property of Angle Congruence: If \( \angle ABC \) is an angle, then \( \angle ABC \cong \angle ABC \).
  o Symmetric Property of Angle Congruence: If \( \angle ABC \cong \angle XYZ \) then \( \angle XYZ \cong \angle ABC \).
  o Transitive Property of Angle Congruence: If \( \angle ABC \cong \angle XYZ \) and \( \angle XYZ \cong \angle PQR \) then \( \angle ABC \cong \angle PQR \).

Theorems:
  - **Right Angle Congruence Theorem**: If two angles are right angles, then the angles are congruent.
  - **Congruent Complements Theorem**: If two angles are complementary to the same angle, or congruent angles, then the angles are congruent.
  - **Congruent Supplements Theorem**: If two angles are supplementary to the same angle, or congruent angles, then the angles are congruent.
  - Linear Pair Postulate: If two angles form a linear pair, then the angles are supplementary.
  - **Vertical Angles Theorem**: If two angles are vertical angles, then they are congruent.

When writing a 2-Column Proof, it may be helpful to write justifications for each statement in conditional form. The conclusion condition of each justification must 'mirror' the statement being justified.

Ex) The statement, \( \angle XYZ \cong \angle PQR \) must be justified by a conditional statement with a conclusion referring to congruent angles, such as, “If two angles have the same measure, then they are congruent.” (Definition of Congruent Angles.)

Review the difference between Postulates and Theorems. It is important to note that the proofs of theorems are just like any other proof we write.