<u>Geometry Unit 1 – Notes</u>
<u>Points, Lines, Planes and Angles</u>

Review Concept: simplifying radicals

Syllabus Objective: 1.1 - The student will illustrate the relationships among geometric terms.

Syllabus Objective: 1.2 - The student will use proper notation to name and label undefined and defined terms.

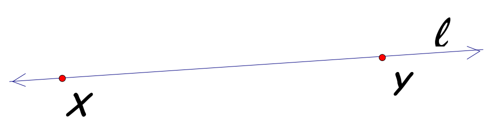
Syllabus Objective: 1.4 - The student will analyze relationships among points, lines, and planes.

<u>Undefined Terms</u>

1. **<u>Point</u>** – has no dimension. It is usually represented geometrically by a small dot. Named by a capital letter. Read: point *A.*

   .*A*

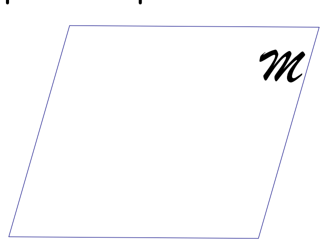
2. **<u>Line</u>** – extends in one dimension indefinitely. It is usually represented by a straight line with arrowheads to indicate that the line extends without end in two directions. In this course, lines are always straight lines. Named by two points or a scripted lower-case letter. Read: line *XY* or line *YX* or line ℓ.



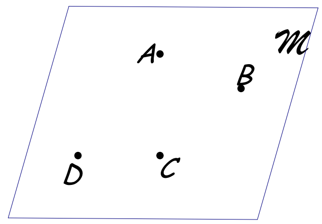
The following are examples of notations for naming lines:

$\overleftrightarrow{XY}$ or $\overleftrightarrow{YX}$.

3. **<u>Plane</u>** – extends in two dimensions. It is usually represented by a shape that looks like a tabletop or wall. You must imagine that the plane extends without end, even though the drawing of a plane appears to have edges. It is a flat surface, has no thickness, and is named by a capital script letter. Read or written as: plane *M*.



Some planes are represented and named with points located on that plane. For example, the plane below contains four identified points. This plane can be named or notated by choosing any three non collinear points. EX: Plane *ABC* or plane *BCD* or plane *ACD.*



The three undefined terms; point, line and plane, are the basis of Geometry. Although they are undefined, students must have an understanding of the concepts of these terms. They cannot be formally defined because they lack any dimensions.

- A **point** has no width no matter what kind of writing utensil is used to create it.
- A **line** has no thickness but although it is confined to the space allowed in a diagram it must be imagined to continue on forever.
- A **plane** has no height and although the diagrams show boundaries, students should imagine it is a puddle that spreads out flat in every direction, acted upon by a force field that can hold it at any slant.

Space – set of all points.

Collinear points – a set of points that lie on one line.

Coplanar points – a set of points that lie on one plane.
Here are different ways of expressing relationships between points, lines, and planes.

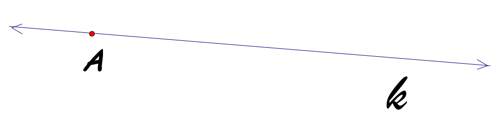
Intersections - a set of all points that geometric figures have in common.

A lies in *k*

A lies on *k*

k contains *A*

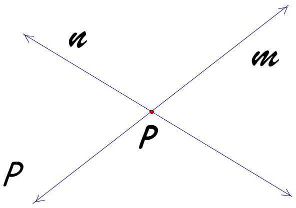
k is drawn through *A*



m and *n* intersect at *P*

m and *n* intersect in *P*

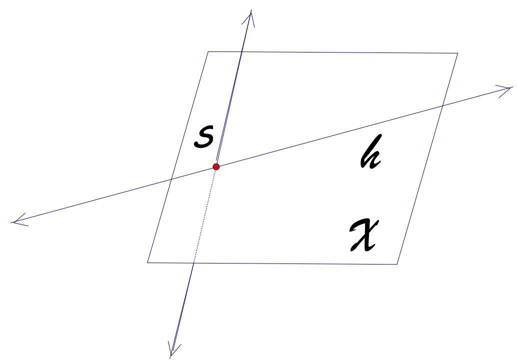
the intersection of *m* and *n* is *P*



S and *k* lie in *X*

X contains *S* and *k*

k intersects *X* at *S*



Terminology

Postulate (axiom) – is a statement (basic assumption) assumed to be true without proof.

Theorem – is a statement that has to be proved.

Corollary – is a special case of a theorem.

Basic Assumptions, the first

Postulate: A line contains at least two points, a plane contains at least three points not all on one line, and space contains at least 4 points not all on one plane.

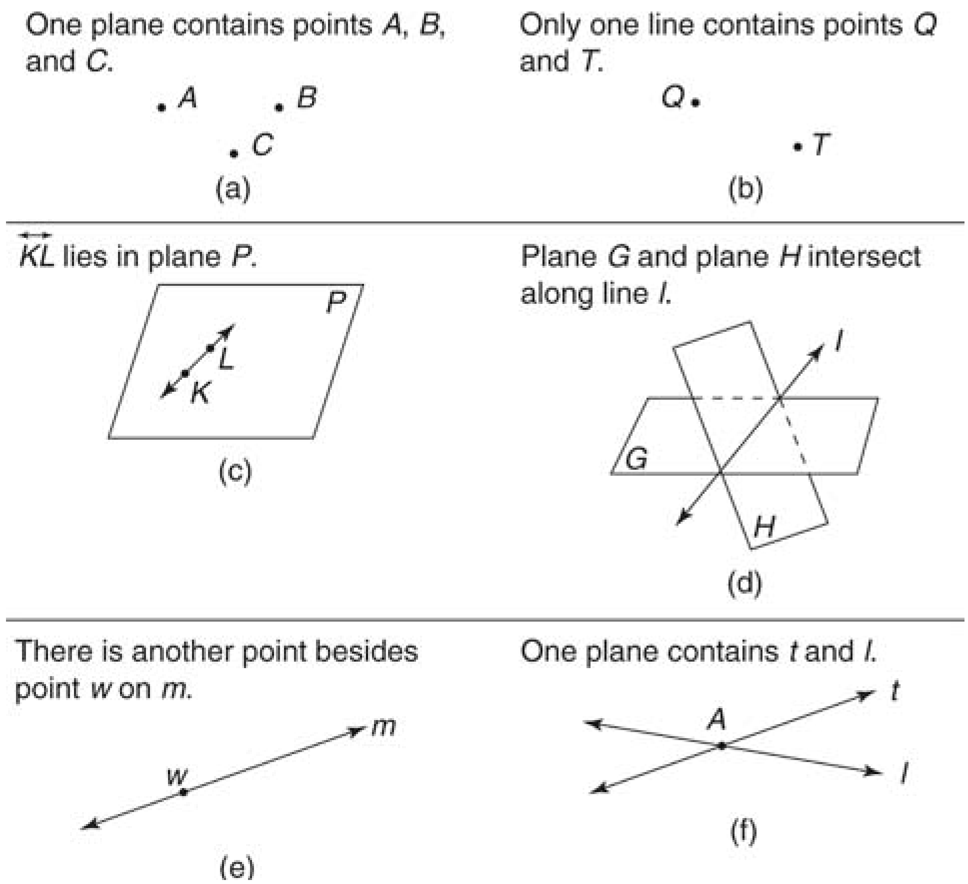
Postulate: Through any two points there is exactly one line.

Postulate: Through any three points not on one line (noncollinear) there is exactly one plane.

Postulate: If two points lie in a plane, then the line joining them is in that plane.

Postulate: If two planes intersect, then their intersection is a line.

In examples a-e, state the postulate or theorem you would use to justify the statement made about each figure.



Solutions:

(a) Through any three points not on one line there is exactly one plane.

(b) Through any two points, there is exactly one line.

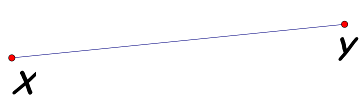
(c) If two points lie in a plane, then the line joining them is in that plane.

(d) If two planes intersect, then their intersection is a line.

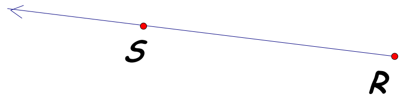
(e) A line contains at least two points…

Subsets of a Line

Segment – Given any two points, *X* and *Y*, segment *XY* is the set of all points consisting of *X* and *Y* and all the points that lie between *X* and *Y*. *X* and *Y* are the endpoints. Denoted by $\overline{XY}$ or $\overline{YX}$.



Ray – Ray *RS*, denoted by $\overrightarrow{RS}$, consists of the initial point *R* and all the points that lie on the same side of *R* as point *S*. (Part of a line that starts at one endpoint and extends forever)



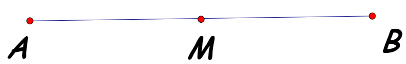
Opposite Rays –($\overrightarrow{SR}$ and $\overrightarrow{ST}$) Two rays with common endpoints that form a line.

Congruent segments – segments with equal lengths. The symbol for length of $\overline{AB}$ is *AB*.

If $AB = XY$, then $\overline{AB} \cong \overline{XY}$

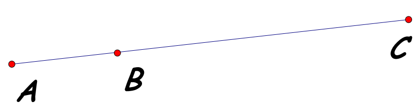
= is used to describe that two numbers are equivalent. $\cong$ is used to describe that two figures are equivalent. They have the same size and shape.

Midpoint of a segment – point *M* is the midpoint of $\overline{AB}$ if *M* lies on $\overline{AB}$ and $AM = MB$.



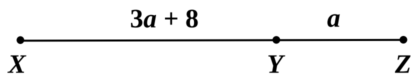
Bisector of a segment – a line, segment, ray, or plane that intersects $\overline{AB}$ at its midpoint is a bisector of $\overline{AB}$.

Segment Addition Postulate: If *B* lies on $\overline{AC}$, then $AB + BC = AC$.



Semester Exam Review (Practice Test 08-09):

In the figure below, *Y* is between *X* and *Z* and $XZ = 40$ cm.



What is the value of *a*?

A. 4

B. 8

C. 12

D. 16

Symbols

$\overleftrightarrow{AB}$ line containing *A* and *B*.

$\overrightarrow{AB}$ ray with endpoint *A*, through *B*.

$\overline{AB}$ segment joining *A* and *B*.

AB length of $\overline{AB}$.

Postulate: For any two points there is a unique positive number called the distance between the points.

Ruler Postulate: The points on a line can be paired with the Real Numbers in such a way that:

a. any desired point can be paired with zero.

b. the distance between any two points is equal to the absolute value of the difference of the numbers paired with those points.

Syllabus Objective: 1.10 - The student will find the midpoint of a segment.

Midpoint Formula

Example: Find the coordinate of the midpoint of a segment that connects:
(10, 0) and (12, 0).

Draw a number line and find the middle of the two coordinates.

Using arithmetic: $\frac{10+12}{2} = \frac{22}{2} = 11$. Therefore the midpoint is (11, 0).

Example: Find the coordinates of the midpoint of a segment with:
$A(2, 3)$ and $B(4, 5)$ as the endpoints.

Draw a number line and find the middle of the line segment connecting 2 and 4. Likewise find the middle of the segment connecting 3 and 5.

Using arithmetic: $\left(\frac{2+4}{2}, \frac{3+5}{2}\right) \rightarrow \left(\frac{6}{2}, \frac{8}{2}\right)$ *or* $(3, 4)$.

The previous examples suggest the midpoint formula – Mdpt $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Semester Exam Review (Practice Test 08-09):

What are the coordinates of the midpoint of the segment joining the points $A(-3,-4)$ and $B(4,2)$?

A. $\left(-3\frac{1}{2}, 3\right)$

B. $\left(-\frac{1}{2}, -1\right)$

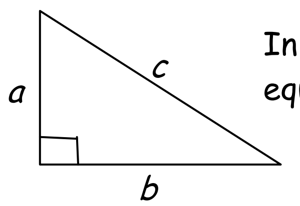
C. $\left(\frac{1}{2}, -1\right)$

D. $\left(\frac{1}{2}, -3\right)$

Syllabus Objective: 1.9 - The student will find the distance between two points.

Distance Formula

Make sure to review the Pythagorean Theorem before deriving the distance formula:
Review:



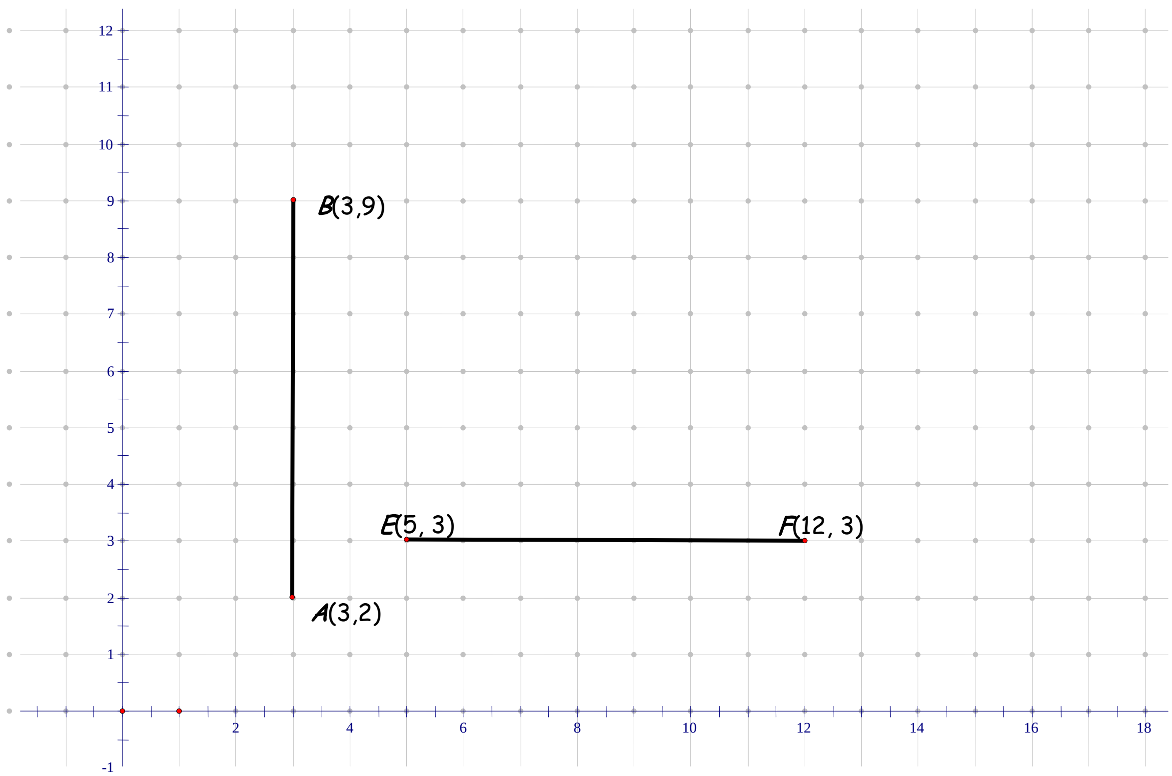
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

$$a^2 + b^2 = c^2$$

Linking the Pythagorean Theorem to the distance formula:

Start by looking at points that lie on vertical and horizontal lines in the coordinate plane:

1. $A(3, 2)$, $B(3, 9)$
2. $E(5, 3)$, $F(12, 3)$

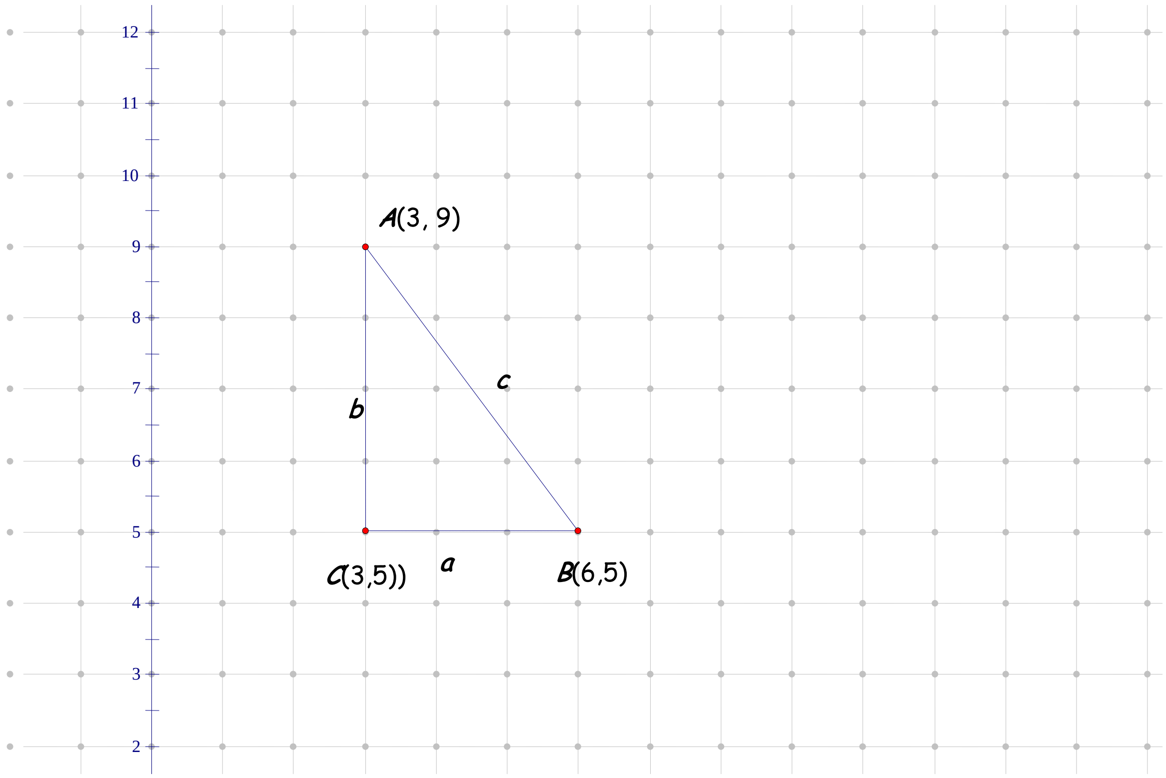


Find the length of each segment:

AB = 9 - 2 = 7 (the difference in the y-values)

EF = 12 – 5 = 7 (the difference in the x-values)

Use the Pythagorean Theorem to find the length of the hypotenuse in a right triangle:



$a^2 + b^2 = c^2$ — {Use Pythagorean triples in examples and practice problems.}

$c^2 = a^2 + b^2$ — (3, 4, 5) (6, 8, 10) (5, 12, 13) (8, 15, 17)

$c = \sqrt{a^2 + b^2}$ — c is always the length of the hypotenuse in a right triangle.

$c = \sqrt{(6-3)^2 + (9-5)^2}$ — a = *BC*. (the difference in y coordinates, Δy)

$c = \sqrt{(3^2 + 4^2)}$ — b = *AC*. (the difference in x coordinates, Δx)

$c = \sqrt{9+16}$ — Link above to the coordinates of points *A* and *B*.

$c = \sqrt{25}$ — Substitute d for c, Δx and Δy for a and b.

$c = 5$ — $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

The distance between two points, (x_1, y_1) and (x_2, y_2) is as follows:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Find the distance between:
(4, 7) and (8, 3).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(8 - 4)^2 + (3 - 7)^2}$$
$$d = \sqrt{(4)^2 + (-4)^2}$$
$$d = \sqrt{16 + 16}$$
$$d = \sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

Semester Exam Review (Practice Test 08-09):

What is the distance between points $A(-2,-6)$ and $B(-2,-3)$?

A. 3

B. $\sqrt{41}$

C. 9

D. $\sqrt{89}$

Semester Exam Review (Practice Test 08-09):

The ΔRST is constructed with vertices $R(-5,2)$, $S(4,1)$, and $T(2,-1)$. What is the length of $\overline{ST}$?

A. $\sqrt{90}$

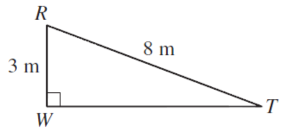
B. $\sqrt{58}$

C. $\sqrt{8}$

D. 2

Sample Nevada High School Proficiency Exam question(s): **Taken from the 2009 Instructional Materials for the NHSPE provided by the Nevada Department of Education**

1. In right triangle *RTW*, shown below, then length of side $\overline{WR}$ is 3 meters (m), and the length of side $\overline{RT}$ is 8 m.

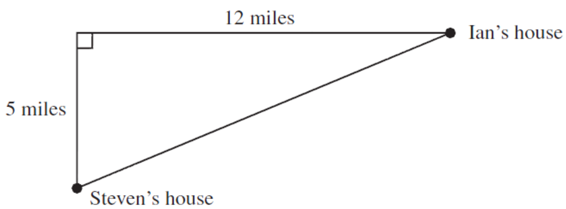


What is the length of side $\overline{TW}$?

A $\sqrt{5}$ m
B $\sqrt{11}$ m
C $\sqrt{55}$ m
D $\sqrt{73}$ m

2. There are two routes that may be used to drive from Steven's house to Ian's house. The routes are described below.
   - Route 1: Drive 5 miles north and then 12 miles east.
   - Route 2: Drive the straight road that goes directly to Ian's house.

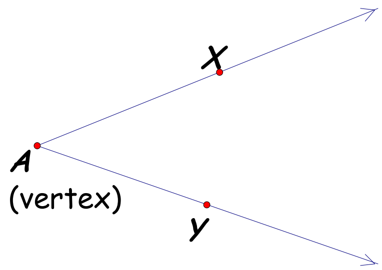
The two routes are shown in the diagram below.



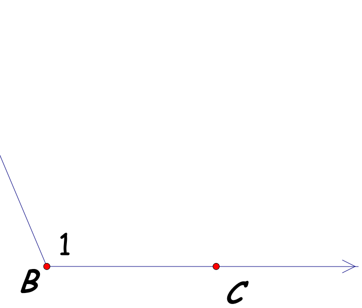
How much longer is route 1 than route 2?

A 4 miles
B 7 miles
C 13 miles
D 17 miles

Angle – is the union of two rays with a common endpoint. The common endpoint is called the *vertex*.



3 Ways to Name an Angle



Examples:

a) Name the angle by its vertex.

$\angle B$

b) Name the angle by three letters, one point on each ray and the vertex being the middle (2 ways).

$\angle ABC$ *or* $\angle CBA$

c) Name the angle by a number written in the interior of the angle.

$\angle 1$

Syllabus Objective: 1.5 - The student will classify an angle by its measure.

Angle Classifications

Acute angle - an angle whose measure is less than 90°.

Right angle - an angle whose measure is 90°.

Obtuse angle – an angle whose measure is greater than 90°, but less than 180°.

Straight angle – an angle whose measure is 180°.

Postulate: For every angle there is a unique number between 0° and 180° called the measure of the angle.

Protractor Postulate: The set of rays which have a common endpoint O can be paired with the numbers between 0 and 180 inclusive in such a way that:

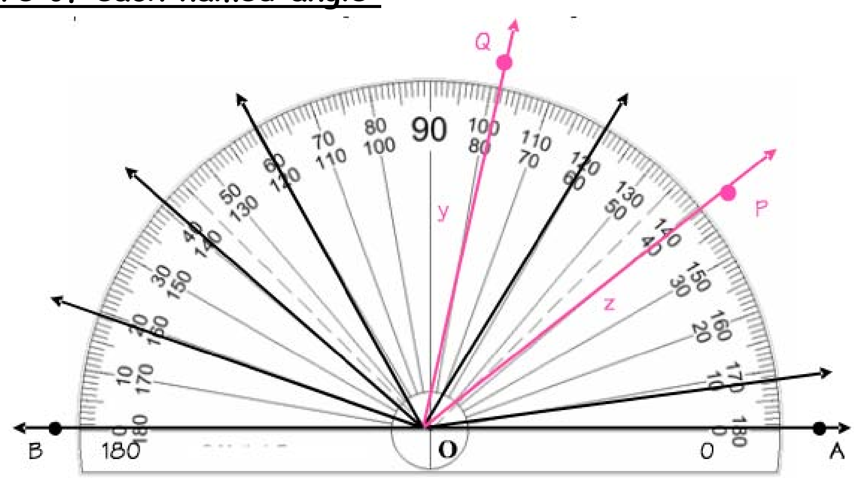
a. one of the rays is paired with zero and the other is paired with a number between 0 and 180.

b. if $\overrightarrow{OA}$ is paired with x and $\overrightarrow{OB}$ is paired with y, then $m\angle AOB = |x - y|$.

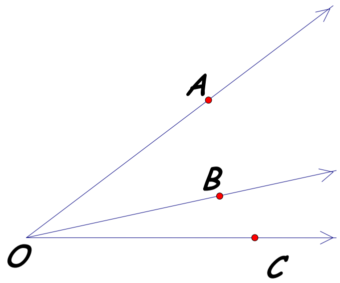
Examples: Find the measure of each named angle:

a) $\angle AOP = 40^\circ$

b) $\angle POQ = 79 - 40 = 39^\circ$



Angle Addition Postulate: If B lies on the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.



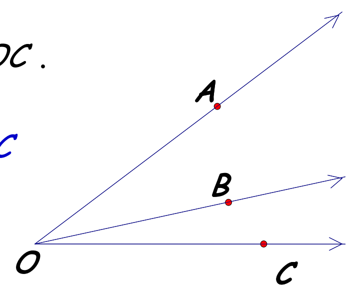
The Angle Addition Postulate indicates that the sum of the parts is equal to the whole. That just seems to make sense.

Examples:

a) Using the answers from the previous set of examples, find the measure of $\angle AOQ$. $m\angle AOP + m\angle POQ = 40 + 39 = 79^\circ$

b) If $m\angle AOB = 30^\circ$ and $m\angle BOC = 15^\circ$, find $m\angle AOC$.

$$m\angle AOB + m\angle BOC = m\angle AOC$$
$$30^\circ + 15^\circ = m\angle AOC$$
$$45^\circ = m\angle AOC$$

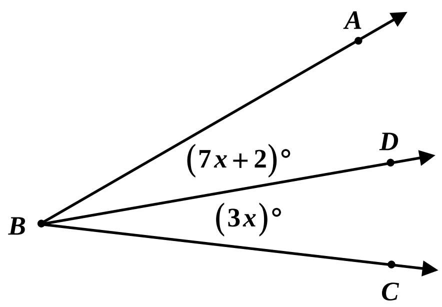


c) Using the diagram from example b, $m\angle AOB = (x+12)^\circ$ is and $m\angle BOC = (x-6)^\circ$. Find the measure of both angles if $m\angle AOC = 46^\circ$.

Note: The parallel "look" of the angle addition postulates and the segment addition postulates.

Semester Exam Review (Practice Test 08-09):

In the diagram below, $m\angle ABC = 42°$.



What is the value of *x*?

A. 2

B. $3\frac{1}{2}$

C. 4

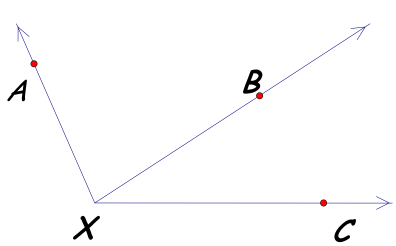
D. $4\frac{2}{5}$

Syllabus Objective: 1.6 - The student will classify pairs of angles.

Syllabus Objective: 1.7 - The student will solve segment and angle problems using algebraic techniques.

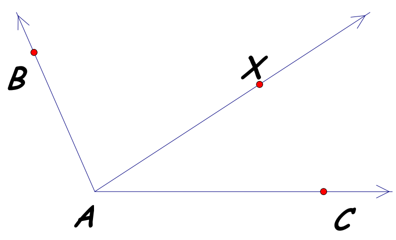
Angle Pairs

Adjacent angles are two angles that have a common vertex, a common side, and no common interior points.



$\angle AXB$ and $\angle BXC$ are adjacent angles. They have a common vertex, *X*, they have a common side, $\overrightarrow{XB}$, and no common interior points.

Angle bisector – $\overrightarrow{AX}$ is said to be the bisector of $\angle BAC$ if X lies on the interior of $\angle BAC$ and $m\angle BAX = m\angle XAC$.



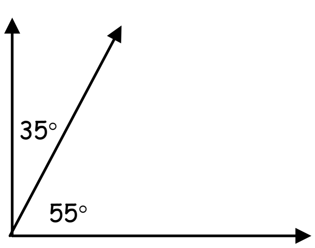
Example: **If** $m\angle BAC = 110^\circ$ **and** $\overrightarrow{AX}$ **bisects** $\angle BAC$**, find** $m\angle XAC$**.**

$$m\angle BAX + m\angle XAC = m\angle BAC$$
$$y + y = 110$$
$$2y = 110$$
$$y = 55$$
$$m\angle XAC = 55^\circ$$

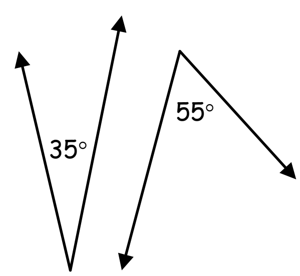
Example: **If** $\overrightarrow{AX}$ **bisects** $\angle BAC$**,** $m\angle BAX = (6x+5)^\circ$ **and** $m\angle XAC = (2x+13)^\circ$**. Find the value of** x**.**

$$m\angle BAX = m\angle XAC$$
$$6x + 5 = 2x + 13$$
$$4x = 8$$
$$x = 2$$

Complementary angles – are two angles whose sum is 90°.



complementary
adjacent

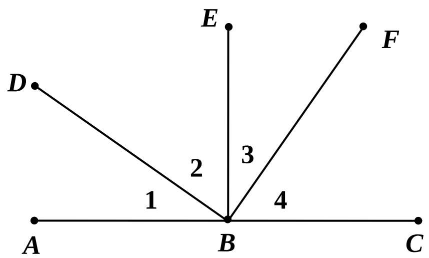


complementary
nonadjacent

Example: If $m\angle A$ = 30°, then the complement of $\angle A$ measures 60°, because (90 – 30 = 60).

Semester Exam Review (Practice Test 08-09):

In the diagram below, $\angle DBF$, $\angle EBC$, and $\angle EBA$ are right angles.



Which best describes the pair of angles: $\angle 1$ and $\angle 4$?

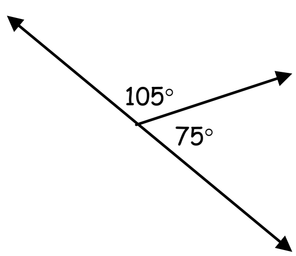
A. vertical

B. adjacent

C. supplementary

D. complementary

Supplementary angles – are two angles whose sum is 180°.



suppplementary
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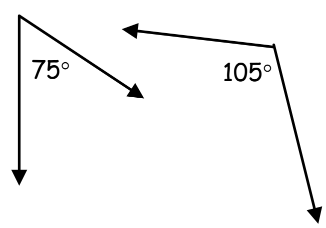


suppplementary
nonadjacent

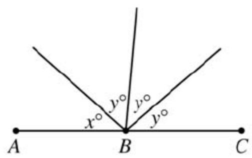
Example: If $m\angle M = 100^\circ$ and If $m\angle M = 100^\circ$, then $\angle M$ and $\angle S$ are supplementary angles, because (100 + 80 = 180).

<u>Example:</u> **<u>Find the value of x:</u>**

If $\angle A$ and $\angle B$ are complementary angles, $m\angle A = 3x°$ and $m\angle B = (2x+10)°$.

$$m\angle A + m\angle B = 90°$$
$$3x + (2x+10) = 90$$
$$5x + 10 = 90$$
$$5x = 80$$
$$x = 16$$

<u>Sample SAT Question(s):</u> **Taken from College Board online practice problems.**



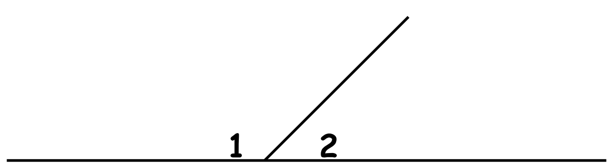
Note: Figure not drawn to scale.

In the figure above, point B lies on $\overline{AC}$. If x and y are integers, which of the following is a possible value of x?

(A) 30
(B) 35
(C) 40
(D) 50
(E) 55

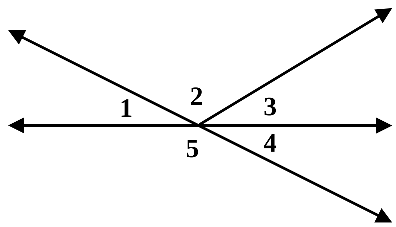
Theorem: If the exterior sides of two adjacent angles lie in a line, then they are supplementary.

These angles, $\angle 1$ and $\angle 2$, are called a **linear pair**. Linear pairs are always supplementary. (Post.)



Semester Exam Review (Practice Test 08-09):

Use the figure below.



Which best describes the pair of angles: $\angle 4$ and $\angle 5$?

A. vertical

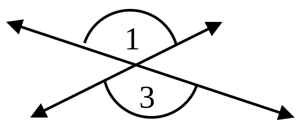
B. adjacent

C. linear pair

D. complementary

Vertical Angles

The mathematical definition of vertical angles is: two angles whose sides form pairs of opposite rays. $\overrightarrow{ST}$ and $\overrightarrow{SR}$ are called opposite rays if S lies on $\overleftrightarrow{RT}$ between R and T.

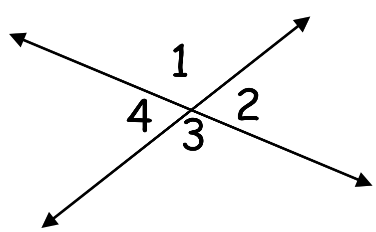


$\angle 1$ and $\angle 3$ are a pair of vertical angles.

Theorem: If two angles from a pair of vertical angles, then they are congruent.

Informal proof

$$m\angle 1 + m\angle 2 = 180^\circ$$
$$m\angle 2 + m\angle 3 = 180^\circ$$
$$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$$
$$m\angle 1 = m\angle 3$$



Syllabus Objective: 1.3 - The student will develop estimation skills using geometric tools.

Students should be given protractors and rulers to practice measuring a variety of items to varying degrees of accuracy. Have students estimate prior to actual measurement.

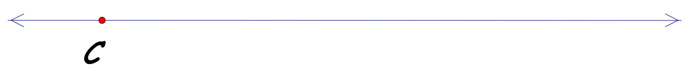
Syllabus Objective 1.8: - The student will use constructions to copy and bisect segments and angles. (Requires supplemental material)

Acceptable methods of construction include patty paper or compass/straightedge constructions.

COPY A SEGMENT

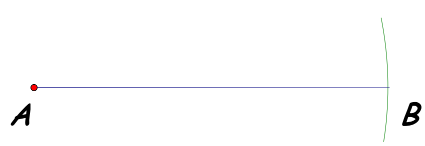
Compass/straightedge:

Copy line segment $\overline{AB}$.



Draw a segment longer than $\overline{AB}$. Establish a point (*C*) on the longer line that corresponds with *A* on $\overline{AB}$.

Place the compass tips on *A* and *B*.



To "measure" length.

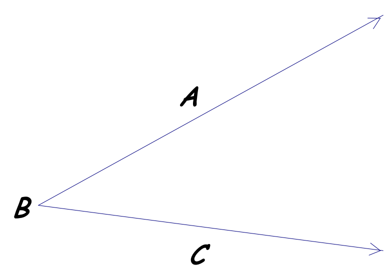
With metal tip at *C*. Transfer length of $\overline{AB}$ to longer line.
Label *D*. $\overline{CD} \cong \overline{AB}$.

Patty Paper:

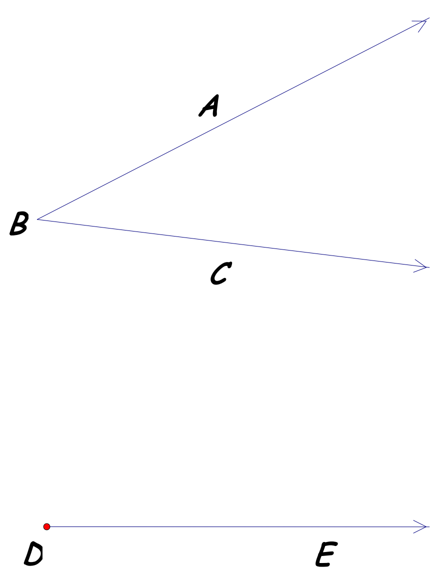
Utilize the "tracing paper" quality of the paper to trace the segment from one paper to another.

COPY AN ANGLE

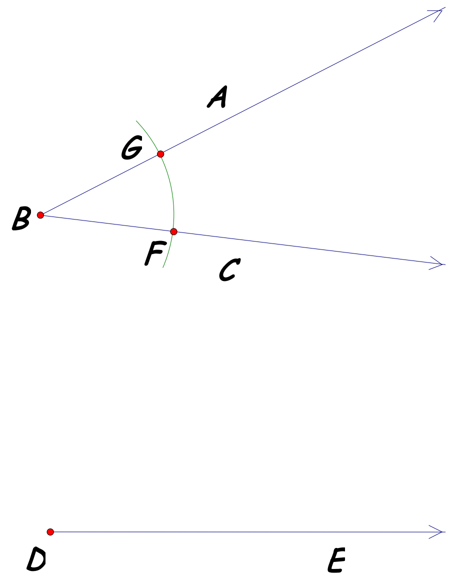
Compass/straightedge:



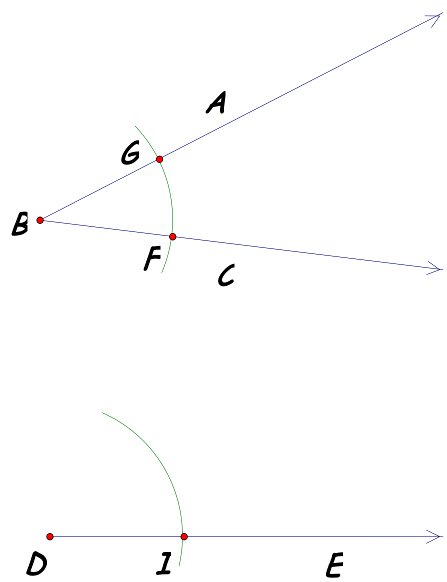
Construct an angle congruent to $\angle ABC$.



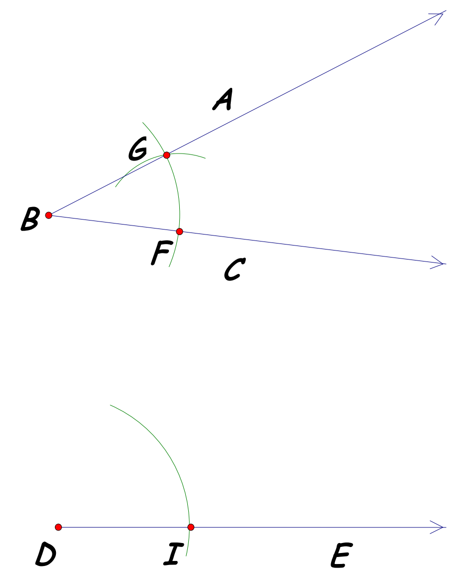
Draw a ray. $\overrightarrow{DE}$.



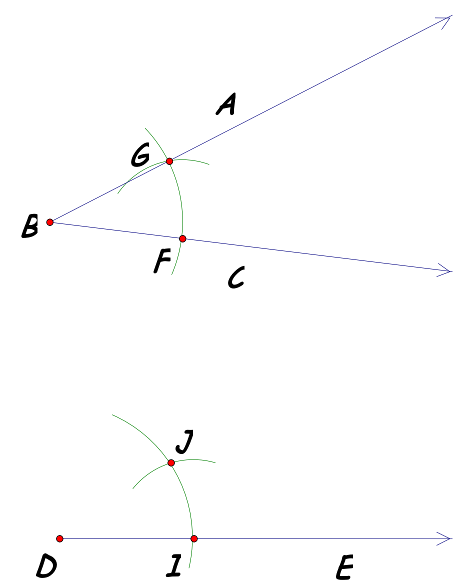
Using ***B*** as a center, make an arc which intersects $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Label the intersections ***F*** and ***G***.



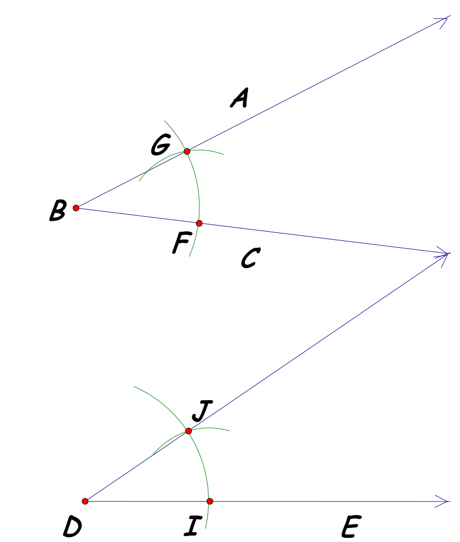
Using ***D*** as a center, mark an arc with the same radius as before, corresponding to $\overline{FG}$. Label the intersection ***I***. Make sure your arc is longer than the angle is wide.



Using *F* as a center, “measure” $\overline{FG}$ with the compass.



Using *I* as a center, and $\overline{FG}$ as a radius, make an arc intersecting the previous arc, label the intersection *J*.



Draw $\overrightarrow{DJ}$. $\angle ABC \cong \angle JDI$.

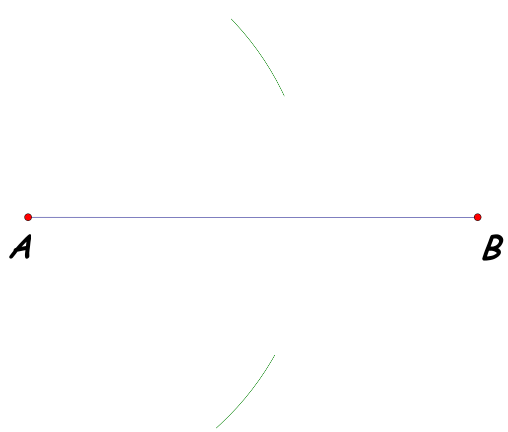
Patty Paper:

Utilize the "tracing paper" quality of the paper to trace the angle from one paper to another.

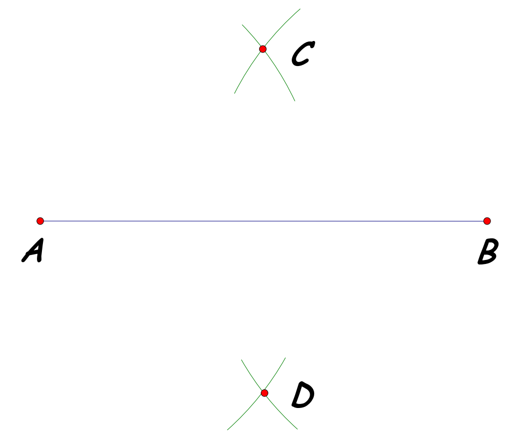
BISECT A SEGMENT

Compass/straightedge:

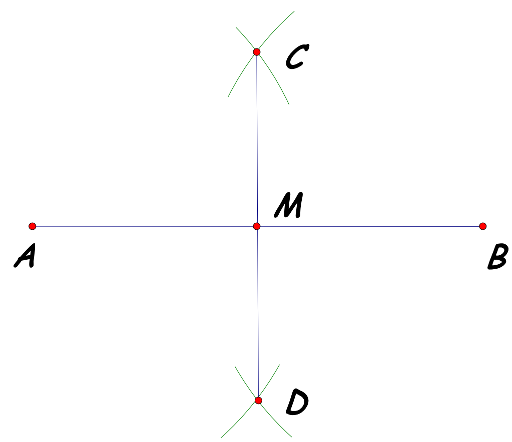
Bisect segment $\overline{AB}$.



Place metal compass tip on ***A***. Using a radius greater than half the length of the segment, draw arcs above and below $\overline{AB}$.



Using the same radius, and ***B*** as the center, draw arcs which intersect the first two arcs. Label the intersections ***C*** and ***D***.



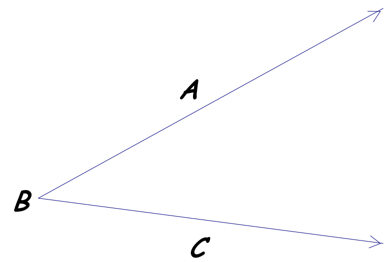
Draw $\overline{CD}$. Label the intersection point ***M***. $\overline{AM} \cong \overline{MB}$ and $\overline{CD} \perp \overline{AB}$. ***M*** is the midpoint of $\overline{AB}$.

Patty Paper:

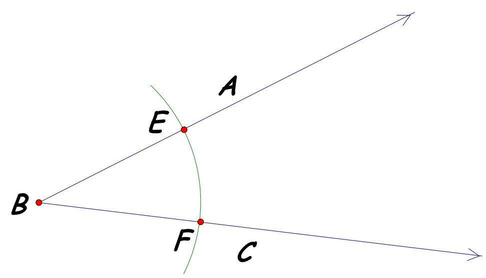
Fold or use a straightedge to create a line segment on a patty paper.
Fold the paper so one endpoint lies on top of the other.
The crease will bisect your line segment.

BISECT AN ANGLE

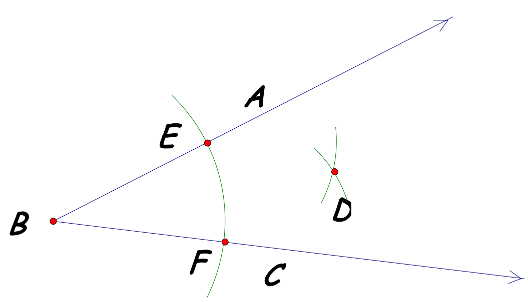
Compass/straightedge:



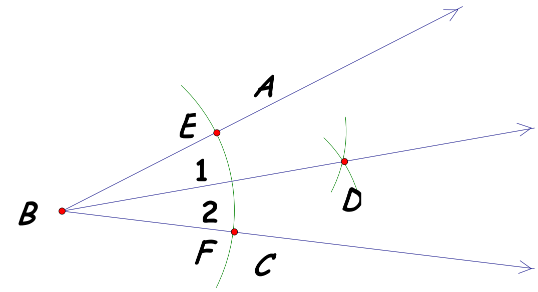
Divide $\angle ABC$ into two congruent angles.



Using *B* as a center, draw an arc which intersects $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at *E* and *F*.



Using *E* and *F* as centers, draw two arcs with equal radii intersecting at a point *D*.



Draw $\overrightarrow{BD}$. $\overrightarrow{BD}$ bisects $\angle ABC$. $\angle 1 \cong \angle 2$.

Patty Paper:

Use a straightedge to draw an angle on a patty paper.
Fold one side of the angle on top of the other.
Make sure your fold also passes through the vertex of the angle.
The crease will bisect your angle.

Semester Exam Free Response Review:

Practice Test (08-09)
Write the step-by-step instructions on how to construct the angle bisector of an angle. Do the construction.