Three-Dimensional Figures

Objectives: (6.19) The student will create a two-dimensional drawing of a three-dimensional figure.
(6.20) The student will create a three-dimensional figure from a two-dimensional drawing.

A solid is a three-dimensional figure that occupies a part of space. The polygons that form the sides of a solid are called faces. Where the faces meet in segments are called edges. Edges meet at vertices.

There are four types of solids that students should be able to identify at the 7th grade level: prisms, cylinders, pyramids, and cones.

A prism is a solid formed by polygons. The faces are rectangles. The bases are congruent polygons that lie in parallel planes.

A cylinder is a solid with two congruent circular bases that lie in parallel planes.

A pyramid is a solid whose base may be any polygon, with the other faces triangles.

A cone is a solid with one circular base.

We further discuss polyhedrons: solids with all faces as polygons. Prisms and pyramids would meet this criterion, while cylinders and cones would not.

The ability to draw three-dimensional figures is an important visual thinking tool. “A picture is worth a thousand words.” Here are some drawing tips:
Rectangular Prism (face closest to you):

Draw the front rectangle. 
Draw a congruent rectangle in another position. 
Connect the corners of the rectangles. 
Use dashed lines to show the edges you would not see. 
Your rectangular prism!

Rectangular Prism (edge closest to you):

Cylinder:

Pentagonal prism:

Cone:
Hexagonal Pyramid:

Another way to look at three-dimensional figures is to look at a net. A net is an arrangement of two-dimensional figures that can be folded to make three-dimensional figures. This will take the student from two-dimensional to three-dimensional. Also have students start working the other way: start with a three-dimensional solid, like a box, and see if they can draw what it would look like if it was “unfolded” and laid flat. Then try a cylinder (an oatmeal cereal container works well!).

The following websites will give you more resources:

- [http://www.mathsisfun.com/platonic_solids.html](http://www.mathsisfun.com/platonic_solids.html) gives you printable nets for the platonic solids, shows figures rotating
- [http://britton.disted.camosun.bc.ca/jbpolytess.htm](http://britton.disted.camosun.bc.ca/jbpolytess.htm) printable nets, tessellated in full color
- [http://www.mathsnet.net/geometry/solid/index.html](http://www.mathsnet.net/geometry/solid/index.html) interactive, allows you to look at the net and then create the solid from the net
- [http://www.adrianbruce.com/maths/nets/cylinder_net.htm](http://www.adrianbruce.com/maths/nets/cylinder_net.htm) printable net for a cylinder
- [http://www.senteacher.org/wk/3dshape.php](http://www.senteacher.org/wk/3dshape.php) printable nets for many different solids

In addition to drawing solid figures and working with their nets, students are expected to create two-dimensional drawings of three-dimensional figures and create three dimensional figures from a two-dimensional drawing. For these notes and the creating of the practice test and test, we have used Microsoft Word. Choose Insert → Shapes → then choose the cube in the Basic Shapes section. You are then able to stack and build almost any 3-D shape of your choosing. Once your figure is built you can “group” the figure to lock the shape. In class you can have students build 3-D figures using wooden cubes, stacking cubes, interlocking cubes or Lego pieces to develop the ability to see the top view, side view and front view.

**Example:** Given the following figure, identify (or draw) the top view, side view and front view.

From the top view, you would see

From the front view, you would see
From the side view, you would see

Example: Given the following figure, identify (or draw) the top view, side view and front view.

From the top view, you would see

From the front view, you would see

From the side view, you would see

Allow students to build and draw figures. As always, begin with very simple figures and allow them to try more complex figures as they are able.

Example: Given the following figure, identify (or draw) the top view, side view and front view.

From the top view, you would see
From the front view, you would see

From the side view, you would see

*Example:* Given the top, side and front views, identify (or draw) the figure.

Top View

Front View

Side View

Answer:

*Example:* Given the top, side and front views, identify (or draw) the figure.

Top View

Front View

Side View

Answer:
Example: Given the top, side and front views, identify (or draw) the figure.

![Top View](image1)

![Front View](image2)

![Side View](image3)

Answer:

Example: Given the top, side and front views, identify (or draw) the figure.

![Top View](image4)

![Front View](image5)

![Side View](image6)

Answer:

Volume

Objectives: (6.3) The student will measure estimate the capacity, volume and surface area of solid figures.

(6.4) The student will measure the dimensions of solid figures to the required degree of accuracy.

(6.5) The student will apply formulas to find the volume of solid figures.

(6.6) The student will solve problems involving volume of solid figures by applying standard formulas.
If you were to buy dirt for your yard, it’s typically sold in cubic yards—that’s describing volume. If you were laying a foundation for a house or putting in a driveway, you’d want to buy cement, and cement is often sold by the cubic yard. Carpenters, painters and plumbers all use volume relationships.

The volume of a three dimensional figure measures how many cubes will fit inside it. It’s easy to find the volume of a solid if it is a rectangular prism with whole number dimensions. Let’s consider a figure 3 m x 2 m x 4 m.

We can count the cubes measuring 1 meter on an edge. The bottom layer is 3 x 2—there are 6 square meter cubes on the bottom layer.

We have three more layers stacked above it (for a total of 4 layers), or $6 + 6 + 6 + 6 = 24$.

Now we can reason that if I know how many cubes are in the first layer (6), then to find the total number of cubes in the stack, you simply multiply the number on the first layer by the height of the stack ($6 \cdot 4 = 24$).

This is a way of finding volume. We find the area of the base (B) and multiply it times the height (h) of the object.

For **prisms and cylinders**, $V = Bh$, where $B$ is the area of the base and $h$ is the height.

Leaving the reasoning to a later course in geometry, we can also state that for **pyramids and cones**, $V = \frac{1}{3}Bh$, where $B$ is the area of the base and $h$ is the height.

For testing at the state level (grade 7) on volume, solid figures may include cubes, rectangular and triangular prisms, cylinders, and triangular and square pyramids.

**Example:** Find the volume of the prism shown.

The bases of the prism are the triangles, so to find the area of the base we will use the formula $A = \frac{1}{2}bh$. The height will be the distance between the two bases (4). We have:

\[ A = Bh \]

\[ A = \frac{1}{2} \times (8)(6)(4) \]

\[ A = 96 \]
The volume of the triangular prism is 96 cubic meters.

Example: Find the volume of a chocolate cake that has a diameter of 24 cm and a height of 14 cm. Use \( \frac{22}{7} \) as an approximation for \( \pi \).

\[ V = Bh. \text{ Our base is a circle, so we will need the radius. The radius is one-half the diameter so } r = \frac{1}{2}(24) \text{ or } 12. \text{ We would now have} \]
\[ V = Bh \]
\[ V = \left( \frac{22}{7} \right)(12)^2(14) \]
\[ V = 6336 \]

The volume of the chocolate cake is 6336 cubic centimeters.

Example: Find the volume of the square pyramid.

\[ V = \frac{1}{3}Bh \]
\[ V = \frac{1}{3}(12)(12)(8) \]
\[ V = 384 \]

The volume of the square pyramid is 384 cubic inches.

Surface Area

Objectives: (6.5) The student will apply formulas to find the surface area of solid figures.  
(6.6) The student will solve problems involving surface area of solid figures by applying standard formulas.

The surface area of a solid is the sum of the areas of all the surfaces that enclose that solid. To find the surface area, draw a diagram of each surface as if the solid was cut apart and laid flat. Label each part with the dimensions. Calculate the area for each surface. Find the total surface area by adding the areas of all of the surfaces. If some of the surfaces are the same, you can save time by calculating the area of one surface and multiplying by the number of identical surfaces.

Remind your students that “nets” are a way to break up these figures into surfaces for which we can easily find the area.
For testing at the state level (grade 7) on surface area, solid figures may include cubes and rectangular prisms only.

Example: Find the surface area of the prism shown.
All surfaces are squares.

\[ \text{Surface Area} = \text{Area of the top + bottom + front + back + side + side} \]

\[ \text{Surface Area} = 49 + 49 + 49 + 49 + 49 + 49 \]
\[ = 294 \text{ cm}^2 \]

The surface area of the prism is 294 cm\(^2\).

Since a cube has 6 congruent faces, a simpler method would look like
\[ \text{Surface Area} = 6 \cdot \text{the area of a face} \]
\[ \text{Surface Area} = 6B \]
\[ \text{Surface Area} = 6bh \]
\[ = 6 \cdot 7 \cdot 7 \]
\[ = 42 \cdot 7 \]
\[ = 294 \text{ cm}^2 \]
Example: Find the surface area of the prism shown. All surfaces are rectangles.

Divide the prism into its parts. Label the dimensions.

<table>
<thead>
<tr>
<th>Bases</th>
<th>Lateral Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>top 15</td>
<td>front 15</td>
</tr>
<tr>
<td>bottom 15</td>
<td>back 15</td>
</tr>
<tr>
<td>2</td>
<td>side 2</td>
</tr>
<tr>
<td>2</td>
<td>side 2</td>
</tr>
</tbody>
</table>

Find the area of all the surfaces.

<table>
<thead>
<tr>
<th>Bases</th>
<th>Lateral Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = bh )</td>
<td>( A = bh )</td>
</tr>
<tr>
<td>( A = 15 \cdot 2 )</td>
<td>( A = 15 \cdot 4 )</td>
</tr>
<tr>
<td>( A = 30 )</td>
<td>( A = 60 )</td>
</tr>
<tr>
<td>( A = bh )</td>
<td>( A = bh )</td>
</tr>
<tr>
<td>( A = 15 \cdot 2 )</td>
<td>( A = 15 \cdot 4 )</td>
</tr>
<tr>
<td>( A = 30 )</td>
<td>( A = 60 )</td>
</tr>
</tbody>
</table>

Surface Area = Area of the top + bottom + front + back + side + side
\[
\text{Surface Area} = 30 + 30 + 60 + 60 + 8 + 8 = 196
\]

The surface area of the prism is 196 cm\(^2\).

Note: Since some of the faces were identical, we could multiply by 2 instead of adding the value twice. That work would look like

\[
\text{Surface Area} = 2(\text{top or bottom}) + 2(\text{front or back}) + 2(\text{side})
\]
\[
\text{Surface Area} = 2(30) + 2(60) + 2(8)
\]
\[
= 60 + 120 + 16
\]
\[
= 196
\]

The surface area of the prism is 196 cm\(^2\). 