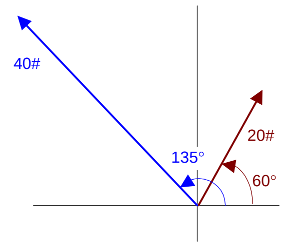
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
January 14, 2002 — High School Edition

This issue of *Take It to the MAT* will focus on *vectors*. We will not be addressing what a vector is, but will be looking at solving problems involving vectors. The applications will be confined to physics, as that is the most common area in which high school students use vectors.

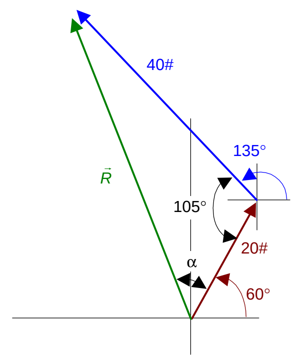
Consider two forces acting on an object. Say one acts with a force of 20 pounds at an angle of 60° with the horizontal and a second of 40 pounds acts at an angle of 135° with the horizontal. What are the magnitude and direction of the resultant force?



The diagrams at right are visual representations of the problem showing the two forces acting on the object and the resultant via vector addition. One of the ways that we teach students to find the resultant force is through the *laws of sines and cosines*. The law of cosines will tell us the magnitude of the resultant, which we will call $\vec{R}$.

$$\left|\vec{R}\right|^2 = 20^2 + 40^2 - 2(20)(40)\cos 105°$$

$$\left|\vec{R}\right| = 49.133\ldots$$



The resultant force has a magnitude of about 49 pounds. Now to find the angle at which the resultant acts, we will use the law of sines.

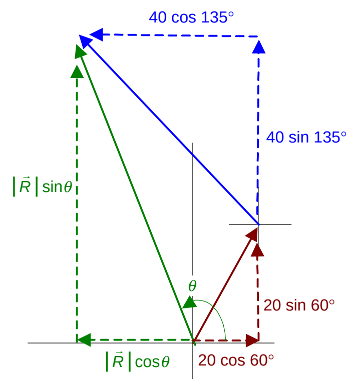
$$\frac{\sin\alpha}{40} = \frac{\sin 105°}{49.133\ldots}$$

$$\alpha = 51.847\ldots°$$

α is about 52°, and the angle at which the resultant acts is α + 60° or about 112°.

Another method is to break each of the vectors into horizontal and vertical components. The components of the resultant are the sums of the corresponding components of the two vectors.

Vector	Horizontal component	Vertical component
20# @ 60°	20# cos 60° = 10#	20# sin 60° = 17.32…#
40# @ 135°	40# cos 135° = –28.28…#	40# sin 135° = 28.28…#
Resultant	–18.28…#	45.60…#



The resultant's magnitude can now be calculated using the Pythagorean Theorem and the direction using the tangent function.

$$\left|\vec{R}\right| = \sqrt{(-18.28\ldots)^2 + (45.60\ldots)^2}$$

$$\left|\vec{R}\right| = 49.133\ldots$$

$$\tan\theta = \frac{45.60\ldots}{-18.28\ldots}$$

$$\theta = -68.152\ldots°$$

However, since we know the resultant is in the direction of the 2nd quadrant, not the 4th, θ is actually –68.152…° + 180° = 111.847…°. Thus, the resultant vector has a magnitude of about 49 pounds in the at approximately 112°.

Which method is better? The law of cosines method requires determining the measure of a couple of other angles before we can get a solution. If a third vector were introduced, the law of cosines becomes more trouble than it's worth. One would have to find the resultant of two vectors, then resolve that answer with the third vector. What a pain! But, solving by breaking vectors into components *always* works, and is efficient regardless of how many vectors are involved. When students take physics, where analysis of vectors is a big topic, their lives will be much easier when approaching vector problems from a component point-of-view.