# TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Southern Nevada Regional Professional Development Program

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In this final issue of *Take It to the MAT* for the 2002–2003 school year, we'll tackle the question posed at the end of the April issue. How do we find the height of a distant object if the observers are not on a level plane? We'll again approach this as the problem of finding the maximum height attained by a model rocket.

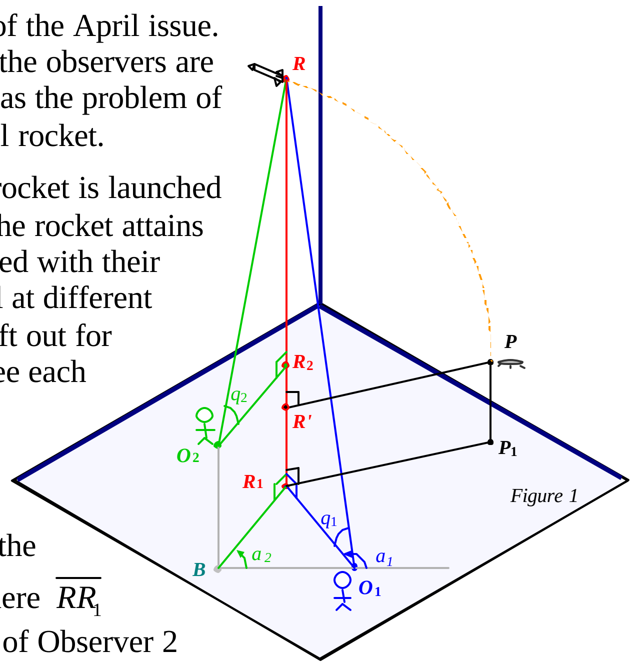

*Figure 1*

*Figure 1* at right shows the situation. A model rocket is launched from a pad and we want to know what altitude the rocket attains with respect to the pad. The two observers, armed with their measuring equipment, and the launch pad are all at different elevations. (The contour of the land has been left out for clarity, but we assume that both observers can see each other, the rocket, and the launch pad.)

We'll use the horizontal plane at the elevation of Observer 1 as our frame of reference and construct $\overline{RR_1}$, the perpendicular segment from the rocket to the plane. The points $R_2$ and $R'$ are where $\overline{RR_1}$ intersects the horizontal planes at the elevations of Observer 2 and the pad, respectively. We want to know the length of $RR'$, the actual altitude the rocket achieved with respect to the plane of the pad. Angles $q_1$ and $q_2$ are the angles of elevation to the rocket; angles $a_1$ and $a_2$ are the angles of azimuth to the rocket from the reference line $\overleftrightarrow{BO_1}$ as a reference. ($\overleftrightarrow{BO_1}$ is $\overleftrightarrow{O_2O_1}$ projected onto the horizontal plane containing $O_1$.) Remember that we have direct lines of sight between the observers and between each observer and the launch pad, so those distances can be directly measured.

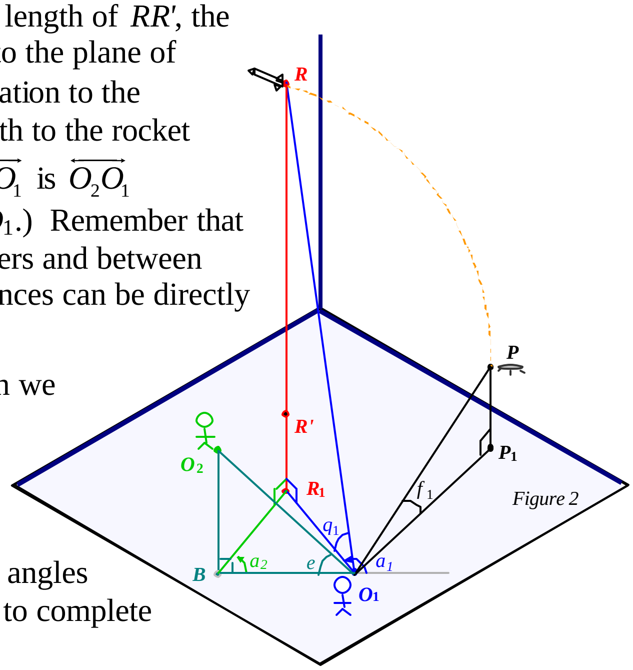

*Figure 2*

Let's cut to the chase and look at the information we really need to find the altitude of the rocket. In *Figure 2*, some information has been added. Angle $f_1$ is the angle of elevation from Observer 1 to the pad and $e$ is the angle of elevation from Observer 1 to Observer 2. Some angles and segments shown in *Figure 1* are not needed to complete the problem and have been deleted.

We want to know the distance $RR'$. We'll start with the fact that $RR' = RR_1 - R_1R'$. (1)

$R_1R'$ is equal to $P_1P$, since $R_1R'PP_1$ is a rectangle. (See *Figure 1*) Triangle $PP_1O_1$ is a right triangle, thus $R_1R' = P_1P = O_1P\sin f_1$. Making a substitution into equation (1), we get $RR' = RR_1 - O_1P\sin f_1$. (2)

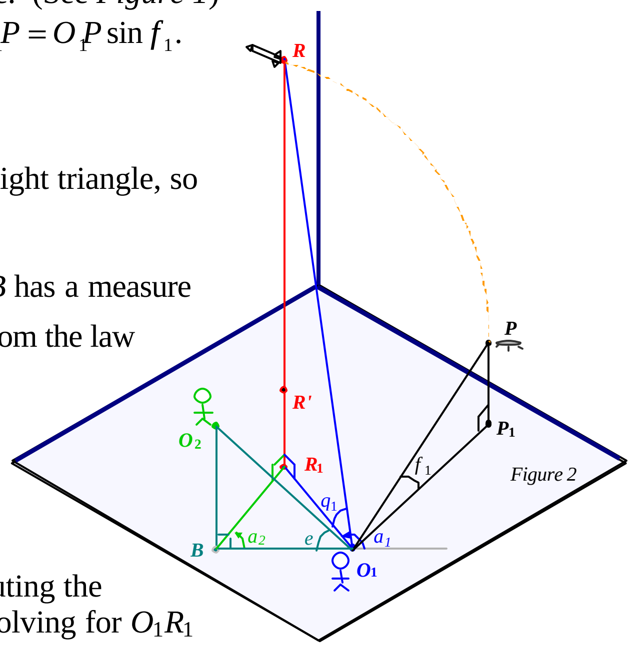

Figure 2

Now to the matter of $RR_1$. Triangle $O_1R_1R$ is a right triangle, so $R_1R = O_1R_1\tan q_1$. (3)

We need to calculate $O_1R_1$. In $\triangle O_1R_1B$, $\angle R_1O_1B$ has a measure of $180^\circ - a_1$, thus $\angle O_1R_1B$ measures $a_1 - a_2$. From the law of sines, $\dfrac{\sin a_2}{O_1R_1} = \dfrac{\sin(a_1 - a_2)}{BO_1}$. (4)

Now we need to find $BO_1$. In right triangle $BO_1O_2$, $BO_1 = O_1O_2\cos e$. (5)

We have all of the pieces of the puzzle. Substituting the right side of equation (5) into equation (4) and solving for $O_1R_1$ we get $O_1R_1 = \dfrac{O_1O_2\cos e\sin a_2}{\sin(a_1 - a_2)}$. Substituting the right side of that into equation (3), we have

$$R_1R = \frac{O_1O_2\cos e\sin a_2}{\sin(a_1 - a_2)}\tan q_1.$$

Finally, substituting into equation (2), we produce $RR' = \dfrac{O_1O_2\cos e\sin a_2}{\sin(a_1 - a_2)}\tan q_1 - O_1P\sin f_1$.

Viola!

Going back to the original problem of finding the height of a distant object there has to be some plane of reference from which to determine the height. In our rocket example, it was the plane of the launch pad. For some other object, perhaps a mountain or building, it could be based upon some fixed location. Alternately, we may want the height based on one of the observers, say Observer 1. In that case, points $P$ and $R'$ are in the same plane as Observer 1, angle $f_1$ has a measure of zero, and the last term in equation $O_1P\sin f_1$ falls out. Essentially, $R_1R' = O_1P\sin f_1 = 0$ because $R_1$ and $R'$ are the same point.

In the March issue we started with a simple problem requiring one right triangle for a solution and gradually increased the scenario's complexity to this one where multiple right and non-right triangles are needed. Students in Precalculus and/or Trigonometry courses should be able to solve these types of problems. They should be encouraged to find their own solutions. When presented with the most complicated situation, as in this issue, students may need to be reminded of the problem-solving strategy to start with a simpler problem.