TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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In the last issue of *Take It to the MAT*, we used triangle trigonometry to find the height of a model rocket at apogee. In deriving the formula we made a grand assumption—that the rocket rises straight up and reaches its peak directly above the launch pad. This is not realistic for any number of reasons, not the least of which is the effect of wind on the trajectory of the rocket. In this issue, we'll look at how to determine the height when the rocket does not rise perfectly vertically.

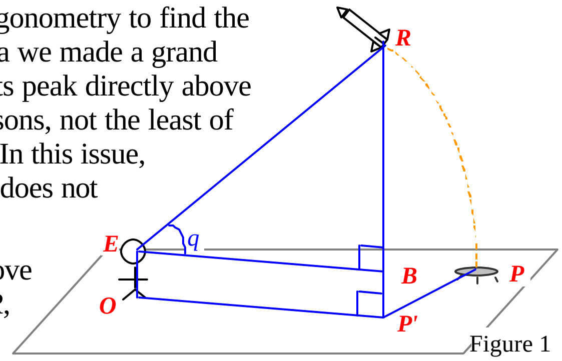

Figure 1

In Figure 1 at right, we see the case where apogee is not above the launch pad. The height of the rocket is clearly $P'B + BR$, and $BR = BE \tan q$, but what is BE? Figure $P'BEO$ is a rectangle, so $BE = P'O$, but we can't determine $P'O$ because we don't know where P' is!

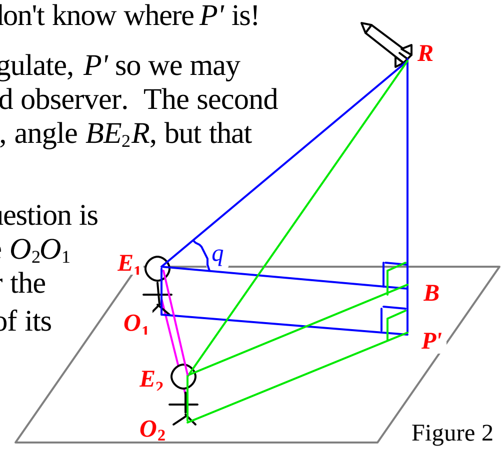

Figure 2

What we need is another observer to help us locate, that is triangulate, P' so we may compute the length of $P'O$. In Figure 2, we have added a second observer. The second observer could also measure his angle of elevation to the rocket, angle BE_2R, but that isn't needed. Observer 2's height O_2E_2 is also irrelevant.

The second observer allows us to form triangle $P'O_1O_2$. The question is what do we know about that triangle? We can directly measure O_2O_1 because the observers' positions are fixed. Now we need either the length of side O_2P' and one angle in $\triangle P'O_1O_2$, or we need two of its angles. It's unlikely that we can determine O_2P' because we don't know where P' is. We'll have to find two angles.

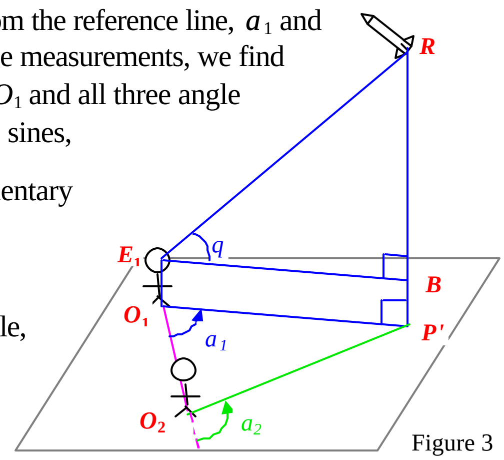

Figure 3

To find the angles we'll need a common frame of reference—we'll use line O_1O_2, as shown in Figure 3. Angles of azimuth from the reference line, a_1 and a_2, are measured for Observers 1 and 2, respectively. Using those measurements, we find $m\angle P'O_2O_1 = 180° - a_2$ and $m\angle O_2P'O_1 = a_2 - a_1$. Knowing O_2O_1 and all three angle measures of triangle, we can now calculate $P'O_1$ using the law of sines,

$\dfrac{P'O_1}{\sin(180° - a_2)} = \dfrac{O_1O_2}{\sin(a_2 - a_1)}$. Recall that the sines of supplementary angles are equal, so $\sin(180° - a_2) = \sin a_2$. Solving for $P'O_1$,

$P'O_1 = \dfrac{O_1O_2 \sin a_2}{\sin(a_2 - a_1)}$. Using the fact that $\triangle BE_1R$ is a right triangle,

$BR = P'O_1 \tan q$. Substituting, $BR = \dfrac{O_1O_2 \sin a_2}{\sin(a_2 - a_1)} \tan q$.

Finally, adding the distance from the ground to Observer 1's eye, O_1E_1, we finally determine the height at apogee to be $P'R = O_1E_1 + \dfrac{O_1O_2 \sin a_2}{\sin(a_2 - a_1)} \tan q$.

This process, once again, assumes that the ground is level. What happens if it weren't?