TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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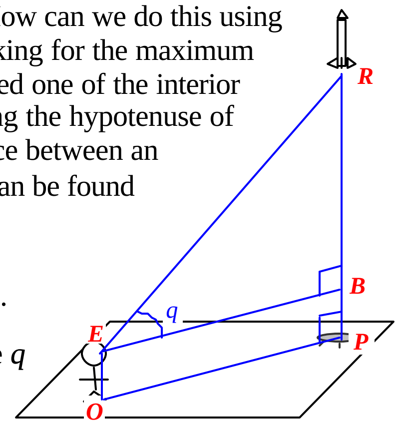
How do we find the height of an object when it is some distance away or there is not a practical physical way to measure it? Trigonometry, of course! We've all seen the classic problems as to how to find the height of a flagpole, or building, or some such object. In this and upcoming issues of *Take It to the MAT*, we will look at this type of problem in the context of finding the apogee of a model rocket.

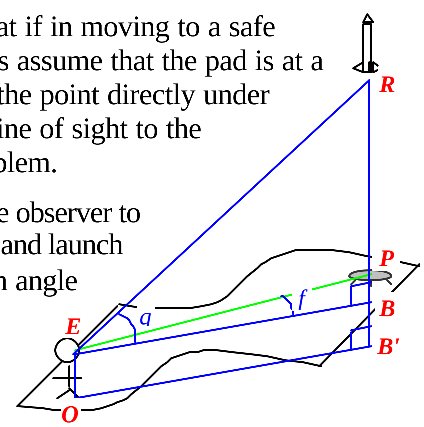
We launch a model rocket and want to determine its height at apogee. How can we do this using trigonometry? The typical process involves a right triangle. We are looking for the maximum height of the rocket *PB*, as seen in the diagram. In order to find it we need one of the interior angles of the triangle and one other side, or both sides. Since determining the hypotenuse of triangle *BER* is unlikely, we'll have to settle for knowing *PO*, the distance between an observer and the launch pad, and one angle. The angle of elevation, q, can be found using a clinometer or similar device.



Right triangle trigonometry tells us that $\tan q = \frac{BR}{BE}$, or $BR = BE \tan q$ *.

However, the height at apogee is actually more than *BR*. Since the angle q was measured to the rocket (*R*) with respect to the horizontal from the eye of the observer ($\overline{BE}$), the distance *BR* is calculated with respect to that level. We want *PR* so we must add *PB*, the distance from the ground to the observer's eye, to get the actual height of the rocket off the ground. Thus, the height at apogee is actually $BE \tan q + PB$*. (*Note that since figure *POEB* is a rectangle, the distance from the observer to the launch pad *PO* is congruent to *BE*, and the height of the observer's eye *OE* is congruent to *PB*.)

We are making a big assumption here in that the ground is level. What if in moving to a safe distance from the launch pad we moved to a different elevation? Let's assume that the pad is at a higher elevation. The distance *BE* cannot be directly measured as *B*, the point directly under the rocket at eye level, is below ground. However, if we still have a line of sight to the launch pad, which is desirable in the interest of safety, there is no problem.



We can directly measure the distance *PE*, the distance from the eye of the observer to the launch pad. Then, we determine the angles of elevation to the rocket and launch pad, q and f, respectively. (If the launch pad is below eye level, f is an angle of depression and is a negative value.) Focusing on triangle *PER* and recalling the law of sines, we can see that $\frac{PR}{\sin(q - f)} = \frac{PE}{\sin(90° - q)}$. (Since triangle *BER* is a right triangle, angle *BRE* is the complement of q.) Recalling that $\sin(90° - q) = \cos q$ and solving for the desired height *PR*, we get $PR = \frac{PE \sin(q - f)}{\cos q}$. Notice in this case that the calculation of the height of the rocket *PR* does not depend on the height of the observer's eye *OE*.

Actually, this is a more general case and works for the first situation, as well. We just didn't measure *PE* in first example.

Finally, this entire discussion is relevant only if the rocket reaches apogee directly above the launch pad. What if that weren't the case? We'll examine that problem in the next issue.