TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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In the last issue of *Take It to the MAT*, we looked at what students must know in terms of trigonometric identities. We continue the discussion in this issue. While the ultimate goal is that students instantly recall the identities, that level of "knowing" is insufficient without conceptual understanding. Connections must be made to previous knowledge and development of the identity must occur. Remember, it's not a matter of *if* a student will forget a particular trigonometric identity, but *when* the student will forget.

In most textbooks, the first sum/difference identity derived is $\cos(A - B) = \cos A \cos B + \sin A \sin B$. The derivation of this identity is not trivial and requires some combination of the law of cosines, the distance formula, calculating the length of a chord, and the Pythagorean trigonometric identities. This is definitely one where students should do the derivation while learning it, perhaps a couple of times, but quick memorization of this one is a plus.

Once $\cos(A - B) = \cos A \cos B + \sin A \sin B$ has been learned, the remaining sum and difference formulas can be quickly derived by substituting $-B$ for B and/or applying the cofunction identities. For example, $\cos(A + B) = \cos(A - (-B)) = \cos A \cos(-B) + \sin A \sin(-B)$. Recalling that $\cos(-x) = \cos(x)$ and that $\sin(-x) = -\sin x$ (cosine is an even function, sine is odd), the identity becomes $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

To get $\sin(A - B)$, remember that $\sin x = \cos(90^\circ - x)$. Thus, $\sin(A - B) = \cos(90^\circ - (A - B))$. Now expand $\cos(90^\circ - (A - B))$ using the cosine difference formula, but watch out! You will have to apply the associative property and expand $\cos((90^\circ - A) + B)$ or you won't get very far. Sin $(A + B)$ again uses the fact that $\cos(-x) = \cos(x)$ and that $\sin(-x) = -\sin x$.

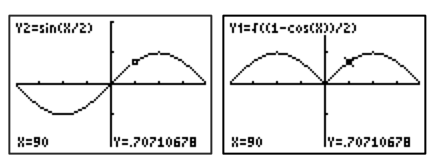
Once the sum identities are known, the double and half-angle identities are not difficult to create.
$\sin 2A = \sin(A + A) = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A$.
$\cos 2A = \cos(A + A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$.
Use of the Pythagorean identity will yield the other two forms of the cos $2A$ identity,
$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.

Half-angle identities come directly from the applying the using the cosine double angle identity.

$$\cos\left(2 \cdot \frac{A}{2}\right) = 2\cos^2\left(\frac{A}{2}\right) - 1 \;\Rightarrow\; 1 + \cos A = 2\cos^2\left(\frac{A}{2}\right) \;\Rightarrow\; \pm\sqrt{\frac{1 + \cos A}{2}} = \cos\left(\frac{A}{2}\right)$$

The sin half-angle identity is derived in the same way applying $\cos 2A = 1 - 2\sin^2 A$.

If a student isn't sure if the identity they need is remembered correctly, a nice method to confirm it is to use a graphing calculator. While it should not be used as a crutch, and an exhaustive search on the calculator is inefficient, the strategy itself is sound. (Note the positive/negative aspect of the half-angle identity in the graphs.)



The whole process of deriving, learning, and memorizing identities is an incremental and methodical one. Half-angle identities come from double angle identities, double angle identities come from sum identities, sum identities come from difference identities that come from other trigonometric properties. Almost all tend to use cofunction and odd/even function identities. The point is that memorization is the ultimate goal, but cannot be the sole manner in which students "know" the identities.