TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Southern Nevada Regional Professional Development Program
January 2003 — High School Edition

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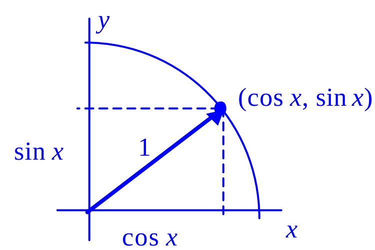
With this issue of *Take It to the MAT*, the Regional Professional Development Program begins its fourth year of providing teachers with a periodical addressing mathematics instruction. We hope you find the topics interesting and useful. —Eds.

Calculus teachers have long maintained that their students would do much better in the course if the students knew trigonometric identities. Precalculus and trigonometry teachers accept this notion as an axiom and do their level best to get students to learn the identities. The heart of the matter is in what it means to “know” the identities. Do they have to be memorized? Or is it simply enough to be able to derive them when needed? The short answer is both.

There are various levels of “knowing”. One level is memorization for instant recall. When working a calculus problem, it is inefficient to be looking up identities in the back of a book. More critically, what would a student do on the Advanced Placement Calculus Examination if an identity were unknown? Probably nothing. Efficiency and performance demand memorization.

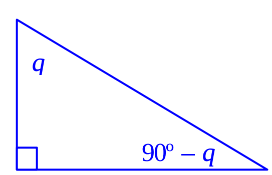
Another level of knowing is the ability to derive the identities using understanding of and linkage to previously learned trigonometric concepts. It is not a matter of *if* a student will forget a particular trigonometric identity, but *when* the student will forget. Without a conceptual foundation, a student that cannot remember the exact details of an identity is sunk. But, with groundwork laid, the student could re-derive the formula if need be.

The most fundamental of trigonometric identities is the Pythagorean identity $\sin^2 x + \cos^2 x = 1$. It’s not a big leap for most students to see from where this originates. A few connections to how the trigonometric functions are defined, the unit circle, and the Pythagorean theorem, and viola! We have our identity. This is not a difficult identity to memorize, either.



The other Pythagorean identities cause some hang-ups for students. Some teachers even have trouble remembering how $\sec^2 x$, $\csc^2 x$, $\tan^2 x$, $\cot^2 x$, and 1 go together. However, re-deriving the identities is a trivial matter. For example, dividing each term in the identity $\sin^2 x + \cos^2 x = 1$ by $\sin^2 x$ yields $1 + \cot^2 x = \csc^2 x$. This is, of course, incumbent upon knowing—memorized, in this case—that $\dfrac{\cos x}{\sin x} = \cot x$ and that $\dfrac{1}{\sin x} = \csc x$. Similarly, dividing through $\sin^2 x + \cos^2 x = 1$ by $\cos^2 x$ yields $\tan^2 x + 1 = \sec^2 x$.

The cofunction identities are not difficult to memorize or derive. Linkage to the basic definitions of the trigonometric identities in a right triangle and we’re there. It’s not hard to see that $\sin(90^\circ - q) = \cos q$. Furthermore, students better remember these if they think of the “co” words. That is, ***co***functions of ***co***mplimentary angles are ***co***ngruent.



Next time, we’ll look at the real toughies—sum, difference, double, and half-angle formulas.