TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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With this issue of *Take It to the MAT*, the Regional Professional Development Program begins its third year of providing teachers with a periodical addressing mathematics instruction. All editions of *Take It to the MAT*—elementary, middle school, and high school—for 2001–2002 will be published tri-weekly. We hope you find the topics interesting and useful. —Eds.

"You can't add apples and oranges." "You can't mix oil and water." "You can't add pink chickens and black cows."

These are just a few of the statements teachers make to students to try to help them understand how to combine like terms. When first confronted with an expression to simplify such as $2x + y$, responses incorrectly include $3x$, $3y$, $3xy$, and $2xy$. Later, $2x + x^2$ mistakenly becomes $2x^2$, $2x^3$, $3x^2$, or $3x^3$. Yet we give students vague metaphors to make the point that these are wrong answers. Instead, we must show them *why* we can simplify an expression by combining *like terms*—and what we *mean* by *like terms*.

Students must initially understand that in the expression $2x + y$, we may not know what x and y are. It may be part of a formula into which we will substitute values for x and y when they are known. Students learn formulas throughout their education and can freely relate to them.

If students are not convinced—or don't understand—that an expression such as $2x + y$ cannot be simplified to something like $3xy$, have the students choose values for x and y, then evaluate. If $x = 6$ and $y = 9$, then $2x + y = 2(6) + 9 = 21$ and $3xy = 3(6)(9) = 162$. Clearly, $2x + y \neq 3xy$.

Care must be taken in the choice of values for x and y. If a student were to choose $x = 1$ and $y = 1$, then $2x + y = 2(1) + 1 = 3$ and $3xy = 3(1)(1) = 3$. Now, $2x + y = 3xy$, but only in this rare case. This is a good time to remind students that in order to make a general statement in mathematics, such as $2x + y = 3xy$, it must be true for *every* possible set of values for x and y.

When simplifying expressions like $2x + x^2$, students sometimes incorrectly see $2x$ and x^2 as like terms since both contain x. The same substitution technique can be used to verify that $2x + x^2 \neq 3x^2$, or $2x + x^2 \neq 3x^3$. If $x = 5$, then $2x + x^2 = 2(5) + (5)^2 = 35$. Yet, $3x^2 = 3(5)^2 = 75$ and $3x^3 = 3(5)^3 = 375$. Obviously $2x + x^2 \neq 3x^2$ or $3x^3$.

Once again, if a student were to choose $x = 0$, a true statement would be made. The point that some values will work and some won't is very important. This, by the way, is frequently present in the quantitative reasoning sections of standardized tests like the SAT. It also helps to build a foundation for solving equations such as $2x + x^2 = 3x^3$.

One other connection to like terms is looking at addition/subtraction of measurements. If one were to add 2 feet and 1 inch, the result is 2 feet (plus) 1 inch. It is not 3 feet or 3 inches or 3 foot-inches. We may convert one unit to the other and then combine the measures, just as we may combine the terms $2x$ and y if we knew how to write x in terms of y or vice-versa. However, such substitutions are unique to the particular values of x and y, just as conversions between units are unique to those units themselves.

The metaphors used above are *not* unacceptable. As a matter of fact, they *should* be used to drive home rules about combining like terms. We must, however, avoid using them without numerical justification.