TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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One of the key properties with which students need fluency is the *zero property of multiplication*, sometimes referred to as the *zero product property*. In this issue of *Take It to the MAT*, we will explore that important principle.

Let's begin with a simple exercise: what is $18 \cdot 13 \cdot 21 \cdot 0$? Let's see $18 \cdot 13 = 234$, $234 \cdot 21 = 4914$, and $4914 \cdot 0 = 0$. Oh my, why didn't we see that zero sooner? In truth, we teachers did, but not all students do. Even if they did see the zero, many times they are stuck in an order of operations rut that leads them to believe they *must* multiply from left to right. The first hurdle is getting students to look at the entire exercise first. Then they need to ask themselves what they know about properties that may help them. In this case, if one or more of any number of factors is zero, then the product is zero.

This property begins to show its real value when we look at it from the opposite perspective: if a product is zero, then one or more of the factors is zero. Consider $18 \cdot 13 \cdot 21 \cdot n = 0$. What does n equal? We need not multiply 18, 13, and 21 to simplify the left side of the equation to be $4914n$, solve $4914n = 0$ for n, and finally figure out $n = 0$. We need only see that there are several factors whose product is zero, thus at least one of them is zero. In this case, only n can equal 0.

The next step in looking at the zero property of multiplication is if we have multiple factors that may be zero. For example: if $m \cdot n = 0$, then either $m = 0$, $n = 0$, or both. We can't be sure which one is or is not zero, but if either or both are zero then the equation is true.

We can extend this idea that multiple factors have the potential to be zero if the product is zero. If we had the equation $(n-2)(n+5)=0$, the product of the two factors $(n-2)$ and $(n+5)$ is zero, so one or the other must be zero. Either $n - 2 = 0$ or $n + 5 = 0$, thus either $n = 2$ or $n = -5$. There are two possible values of n that make the equation true. Unlike the previous example where both factors m and n could equal zero simultaneously, in this case $(n-2)$ and $(n+5)$ cannot both be zero since there is no single value of n that would make both $n - 2 = 0$ and $n + 5 = 0$.

The reason this is so important is that it is the basis for equation solving techniques. With the major exception of linear equations, we usually manipulate the equation so one side is equal to zero. This is for the singular purpose of taking advantage of the zero product property. We count on the fact that if a variable expression is equal to zero and can be factored, it can be easily solved. If $n^2 + 3n = 10$, then $n^2 + 3n - 10 = 0 \Rightarrow (n-2)(n+5) = 0$, thus $n \in \{-5, 2\}$.

It matters not if we have two factors or twenty; the principle is the same. Even though we don't expect first-year algebra students to solve complex equations of degree three and above, there is no reason that such a student could not determine that if $n(n-2)(n-\pi)(2n+3)=0$, then $n \in \{0, 2, \pi, -\frac{3}{2}\}$.

Lastly, students sometimes make a false generalization with respect to the multiplication property of zero by making up new properties, like the "multiplication property of three". They mistakenly conclude that if the product of two factors is three, one of them is also three. For instance, if $(n-2)(n+5)=3$, then $n - 2 = 3$ or $n + 5 = 3$, and thus $n = 5$ or $n = -2$. By plugging –2 and 5 back in to the original equation we can see that this is not true—kids need to see it too.