TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
Regional Professional Development Program
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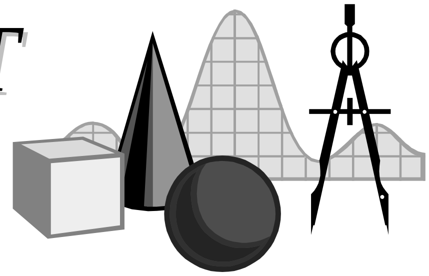
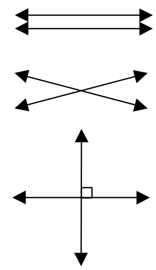


In this final edition of *Take It to the MAT* for the 2000–2001 school year, we'll look at a smorgasbord of misconceptions students sometimes have.



Intersecting vs. perpendicular lines. Students learn that any pair of *lines in a plane* may be described in one of three ways: parallel, intersecting, and perpendicular. These are often shown in diagrams similar to the ones at right. Here's a question to ponder: *Do perpendicular lines intersect?* Of course they do, but because of teaching these concepts in isolation, students come to believe that *only one* of the descriptions can apply. Students must be taught that lines in a plane are either intersect or are parallel, and perpendicularity is a special case of intersecting lines.

"You can't divide by a larger number." Students are repeatedly told that when dividing a quantity into either several groups or into groups of several things, statements like 8 ÷ 24 do not make sense because, "You can't divide a smaller number by a larger number." What if a teacher were to divide 8 chocolate bars equally among 24 students? Each student would receive one-third of a bar, and the corresponding number sentence would be $8 \div 24 = \frac{1}{3}$. We can divide by "larger numbers;" the result is a fraction less than one.

The last three places in a whole number are "the hundreds." When we teach place value, particularly in the intermediate grades, we teach the *periods* of the places. We usually teach the billions, millions, and thousands periods correctly, but we often refer to the last period as the hundreds. This is not the case. The beauty of our place value system is in its repetitive nature. Each period has a hundreds place, tens place, and ones place. Hence, we have one thousands, ten thousands, and hundred thousands. But what we do not have is one hundreds, ten hundreds, and hundred hundreds. The first period to the left of the decimal is composed of one ones, ten ones, and hundred ones.

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
2	5	9	0	1	3	6	8	4

2 dogs plus 3 cats equals 5. Often we have students practice addition facts by asking different ways to form a sum, for instance, 5. This practice is often accompanied by models, through manipulatives or the drawing of pictures. It is often the latter, if not scrutinized, that can create misconceptions. Is two dogs plus three cats equal to five? If it is, five what? Pets? Animals? If we chose to the use the label "pets", then the quantity tells us little about what we actually have, i.e., it does not tell us that we have 2 dogs and 3 cats.

Looking at it another way, what is two-thirds plus three-fourths? Is it five of something? It would be better if we stuck with a single unit in building addition concepts—two dogs plus three dogs is five dogs. Three-eighths plus two-eighths equals five eighths. In measurement, 1 foot plus 4 feet equals five feet. Later in algebra, $2x + 3x = 5x$. By using multiple units in a single example, students are left with the false belief that *anything* can be simplified to get a sum, and that the sum has some meaning.