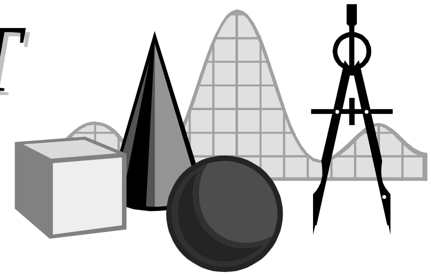
TAKE IT TO THE MAT



A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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As we start this issue of *Take It to the MAT*, grab a sheet of scratch paper and write the down the following numbers: (a) one quadrillion, four hundred thirty trillion; (b) seven hundred eighty-six ten trillionths. Take your time—get another pencil if needed.

OK, let's check your answers: (a) 1.43×10^{15}; (b) 7.86×10^{-11}. Oh, did you answer: (a) 1,430,000,000,000,000; (b) 0.0000000000786? Seems like a lot of writing for a couple of numbers, doesn't it?

Having an alternate method to write very large and very small numbers is the concept behind *scientific notation*. In science and engineering, both miniscule and enormous quantities are commonplace. Writing numbers with oodles of digits is very time consuming and prone to errors. Scientific notation mitigates these two concerns and has some side benefits that will be discussed in a later issue.

Scientific notation is defined as the product of a decimal numeral between one (inclusive) and ten and a power of ten. That is, the number is of the form $a\times10^n$, where $1\le a<10$ and n is an integer. The value a is referred to as the *mantissa* and the value of n as the *characteristic*.

Consider the number 425. We can write this in a variety of ways—so should students—as some number times a power of ten. Examine the box at right. Only one of these fits the definition of scientific notation, the one where the mantissa is 4.25. Thus, 425 = 4.25 × 100, or more appropriately 4.25×10^2. It is important that students see 425, or any other number, in a variety of ways. This builds number sense and makes scientific notation more concrete.

425
$= 425\times1$
$= 42.5\times10$
$= 4.25\times100$
$= 0.425\times1000$
6.7
$= 670\times0.01$
$= 67\times0.1$
$= 6.7\times1$
$= 0.67\times10$
0.056
$= 56\times0.001$
$= 5.6\times0.01$
$= 0.56\times0.1$
$= 0.056\times1$

Students should experience the same with numbers between one and ten. For example, the number 6.7. To get the correct mantissa for scientific notation, we would use 6.7 × 1, which is expressed correctly as 6.7×10^0. It is important that we remember $10^0=1$ and that 1 is indeed a power of 10.

Numbers less than one are also important, and how we read them can help in setting up the number for conversion to scientific notation. The number 0.056 is read as "fifty-six thousandths." That is 56 times one one-thousandth. So, 0.056 = 56 × 0.001, but also can be written as 560 × 0.0001 or as 5.6 × 0.01. Written properly, 0.056 = 5.6×10^{-2}.

In exploring various expressions for the same number, students should also see the pattern that multiplying (or dividing) the mantissa by ten corresponds to dividing (or multiplying) the power factor by ten. This is clearly seen in the box and is very important in reinforcing the concept of balance in changing expressions. That is, if one multiplies a number by ten, then divides by ten, that the number is equivalent to the one before even though it may *look* different.

Next time, we'll examine some "recipes" regarding scientific notation and potential calculator pitfalls.