TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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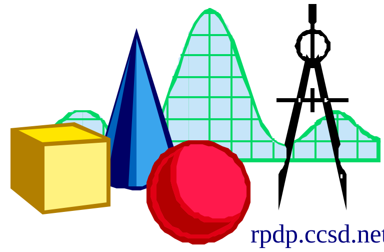

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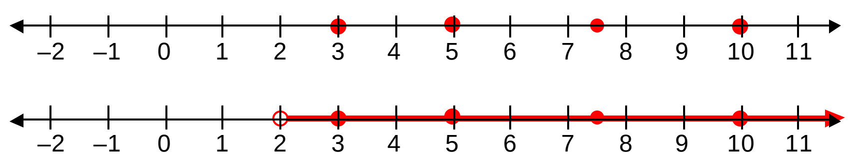
In the October 1, 2001 issue of *Take It to the MAT*, we looked at variables and explored various characteristics of them. While *variable* is the catch-all term for letters used to represent numerical quantities, we learned that variables don't always vary—that sometimes they can only have a single value. In this issue, we'll go a little further, looking at equations and inequalities, what the difference is between them, and how their variables behave.

An *equation* is a mathematical statement in the form $A = B$, where A and B are equal quantities or equivalent variable expressions. They may range from the simple $2 + 3 = 5$ to the complex $x + x^2 + 2x = x^2 + 3x$. Equations may also be *open sentences* that contain variables having an unknown value such as $n + 3 = 5$, or be *formulas* that relate quantities in a particular way such as $A = l \times w$.

In the cases of $2 + 3 = 5$ and $x + x^2 + 2x = x^2 + 3x$, we merely have statements that are always true. The left and right sides of the equations are equivalent. In $n + 3 = 5$, the **only** value of n that makes the sentence true is 2. The variable n doesn't vary from 2 if the statement is to be true. In the formula $A = l \times w$, each of A, l, and w may vary so long as the product of l and w is equal to A. Beyond that, there's not much to say.

We contrast equations with *inequalities*, a mathematical statement in one of the following forms: $A < B$, $A > B$, $A \le B$, or $A \ge B$ where A and B are quantities or variable expressions. Again, they may be simple as in $2 < 5$, or complex as in $x^2 + 2x < 3x + 4$. Like equations, inequalities may be open sentences that contain variables such as $n + 3 > 5$. However, unlike the equation $n + 3 = 5$, the inequality $n + 3 > 5$ has multiple solutions.

Finding the solutions to the inequality need not be difficult or mechanical. Let's ask, "What makes the sentence true?" Well, n could be 5, or 10, or 3, or $7\frac{1}{2}$, or …. (See number line.) It turns out that n could be any of an infinite number of values so long as it is greater than 2. Note that if we only consider whole numbers, it must be 3 or more. If we consider fractions, then anything with a value of 2-and-a-little-bit-more would work, such as $2\frac{1}{1000}$. In this case, n really can vary. Keep in mind that n cannot be 2, because $2 + 3$ is *not* greater than 5. On the number line, we show that two is not included by using an open circle.



Take note that with the equation $n + 3 = 5$, we can give the solution as another equation: $n = 2$. With the inequality $n + 3 > 5$, we can give the solution as another inequality: $n > 2$.

There are multiple ways to express the solutions to inequalities and students should be flexible in their use. For instance, if $n - 3 \le 5$, then we would expect the elementary student to write "$n \le 8$", write and say, "n is less than or equal to 8," write and say, "n is at most 8," and model it (e.g. a number line). The language connection between "less than or equal to" and "at most" is tough at first because "less" and "most" are contradictory. Students can and do understand after many experiences in thinking about the meaning of the words. Similarly, they must make the connections among ≥, "greater than or equal to," and "at least."