TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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Sometime during the elementary grades, usually third, students begin learning their multiplication facts. Most teachers agree that students need to know their facts—products with factors 0–12—as well as they know their own name. Most teachers know that some facts come easier than others; the "ones" and "twos" are memorized more quickly than the "sevens" and "eights."

While we know that knowing and instantly recalling facts is paramount to learning, understanding, and doing higher mathematics, we rarely discuss which fact(s) is/are most important. Is it $2 \times 2 = 4$ or $7 \times 8 = 56$? Have you really ever thought about that? Algebra teachers do and their conclusion is clear.

It may be a stretch to say that the "zeros" multiplication facts are the *most* important, but in algebra they are *very* important. So what makes them so special? It's the fact that any number multiplied by zero is zero. It matters not whether it be 1×0 or $1{,}000{,}000 \times 0$, the product is zero. As a matter of fact, if we want to know the product of several factors, if one is zero the answer is clear. This fact is often used in math contests where students are asked to find the product of something like $20 \times 15 \times 10 \times 5 \times 0$. Students who have trouble with this type of problem usually have not internalized (a) that we can multiply these factors in any order we wish, and (b) that if any of the factors is zero then so is the product. It's not that kids can't multiply 20 by 15, then that product by 10, then that one by 5 to get 15,000, but it's all the time they took to do that just to get 0 in the end.

OK, so why is multiplication by zero so important in mathematics? When finding the solutions to equations, the typical strategy is to manipulate the equation so that we have an expression equal to zero. When an expression is equal to zero, and it can be written as the product of several factors, we can determine what those factors are.

For instance, if $n \times 3 = 0$, then $n = 0$. If $n \times n = 0$, then $n = 0$. There is nothing else n could be in either case. No other number could make the equation true.

Let's build on that first one a little. What if $(n-1) \times 3 = 0$? The left side of the equation is the product of two numbers: 3 and $(n-1)$. (Wait a minute! How is $(n-1)$ a number? Well, if n is a number, one less than it is also a number. But we digress.) Anyway, if 3 times $(n-1)$ is equal to zero, then $(n-1)$ is equal to zero. If $n - 1 = 0$, then $n = 1$.

Now, the previous examples are pretty basic in the grand scheme of algebraic equation solving, and it is not recommended that elementary students solve equations like those above. The point is that the *zero property of multiplication*—that if one or more of several factors is zero, the product is zero—is a very important property in mathematics. While it may seem very trivial at first, it is a key concept that students need to be successful in algebra.