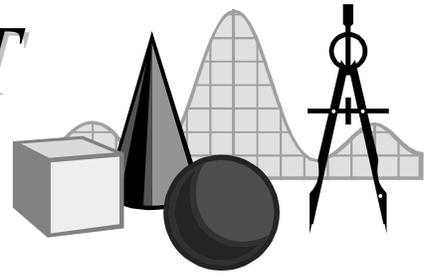


TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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There is a branch of mathematics known as *number theory* that is the study of the properties of numbers, particularly factoring, divisibility, and prime numbers. In this edition of *Take It to the MAT*, we will focus on two topics in number theory: factor/multiple/divisibility and prime/composite numbers. (Note: When the term *number* is used in this issue, it refers specifically to *whole number*.)

Factor: A number whose product with some number is the given number. For example, 3 is a *factor* of 12 because $3 \times 4 = 12$. Additionally, 1, 2, 6, and 12 are also factors of 12 because $1 \times 12 = 12$ and $2 \times 6 = 12$. A number's factors must necessarily be less than or equal to the given number.

Divisible: If a number is a factor of a given number, then we say that the given number is *divisible* by the first. That is, since 3 is a factor of 12, 12 is divisible by 3. The number 12 is also divisible by 1, 2, 4, 6, and 12.

Multiple: A number that is the product of a given number and another number. For example, 12 is a *multiple* of 2 because $12 = 2 \times 6$. Given that last statement, 12 must also be a multiple of 6. Furthermore, 12 is also a multiple of 1, 3, 4, and 12. A whole number's multiples are necessarily greater than or equal to the given number.

Prime Number: A number, which is divisible by no numbers except itself and 1. For example, the number 23 is *prime* because it is only divisible by itself and 1. By definition, 1 is *not* prime; neither is 0 a prime. (See the March 3, 2000 issue of *Take It to the MAT* to see why 0 is not divisible by itself.)

Composite Number: A whole number that is divisible by numbers other than itself and 1. For example, 25 is a *composite* number because it is divisible by itself, 1, and 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Let's tie some things together. How many factors does a composite number have? The answer is *more than two*. A composite number must be divisible by (have factors of) 1, itself, and some other number; i.e. it must have at least three factors.

A prime number is a multiple of how many numbers? Answer: *only two*. A prime number has only two factors, itself and 1. Thus, it is a multiple of only itself and 1. Remember, 1 is not prime!

Here's one for you. Since all even numbers are divisible by themselves, 1, and 2, can any even numbers be prime? Only one, er, make that two. The only even prime number is 2; it has only two factors—itsself and 1.

One final question to think about: What numbers are both multiples and factors of themselves?