TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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Regional Professional Development Program
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Let's begin this edition of *Take It to the MAT* with a couple of exercises:

① $0 \div 6$

② $6 \div 0$

So, did you get zero as the answer to both? This issue will focus on the number *zero* and its behavior in the binary operation of division.

First, let's quickly review the concept of division. What does $6 \div 2$ really mean? Basically, take six objects and place them in groups of two as in *Figure 1*. The *quotient* of three represents the number of groups of two. (Quotient is derived from a Latin word for "how many times.") Thus, $6 \div 2$ asks, "How many groups of two objects can be created from six objects?"

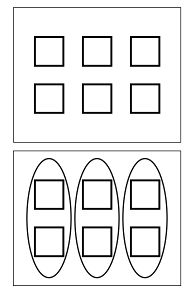

Figure 1

Referring back to the first exercise above, how many groups of six objects can be made from zero objects? No kidding—none! If we have nothing to begin with, we can't very well make any groups of six either, can we?

Now, here's an often-missed connection. Six objects divided into groups of two yields three groups. But, what about reversing the thinking? Three groups of two objects is six objects total. The connection between division and multiplication is strong and important for kids to see and understand. We teach kids to check division by using multiplication. $6 \div 2 = 3$ connects to $3 \times 2 = 6$. So, going back to the original problem, zero groups of six is zero objects. $0 \div 6 = 0$ connects to $0 \times 6 = 0$.

How about exercise two, $6 \div 0$? The knee-jerk answer is zero. Let's investigate.

Using *Figure 2*, take those six objects and divide them into groups of zero. Having trouble? That is because division by zero is meaningless—it cannot be done. The correct mathematical term for division by zero is *undefined*, but grade level will dictate the use of that term.

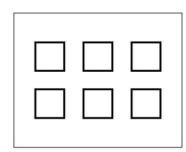

Figure 2

Think about it. First, if we incorrectly assume $6 \div 0 = 0$, then that implies six objects placed into groups of zero gives zero groups. By the connection to multiplication, zero groups of zero objects is six objects total, or $0 \times 0 = 6$. Secondly, what if we replace six by some unknown number like n. If $n \div 0$ has a quotient, say q ($n \div 0 = q$), then our connection implies that $n = q \times 0$. But, we know that regardless of the value of q the product of it and zero must be zero. So, division by zero is a violation of mathematical principles.

One last thing—what is $0 \div 0$? If you have zero objects, how many groups of zero objects can you make from it? One? Two? Twenty? In fact, as many as you want. We say that the quotient of $0 \div 0$ is *indeterminate*—one cannot determine an answer. Grade level will again determine the use of that word. In closing, you are challenged to show, by the connection to multiplication—why is this true?