TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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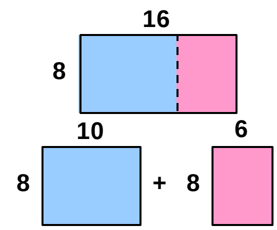
In the December 3, 2001 issue of *Take It to the MAT*, we looked at the associative and commutative properties of addition and multiplication and how they lay the foundation for algebraic thinking. In this issue, we will examine a third operational property—the distributive property.

What is often referred to as the *distributive property* is actually called the *distributive property of multiplication **over** addition.* The phrase *multiplication **over** addition* tells us exactly how the property, which involves **two** operations, behaves.

We often see the distributive property defined in symbols as $a \times (b + c) = (a \times b) + (a \times c)$, followed with examples such as $8 \times (10 + 6) = (8 \times 10) + (8 \times 6)$. The computational advantage of the distributive property is in decomposing numbers into easier values that may be otherwise difficult to multiply mentally. If one could not instantly compute 8×16, one could think of 16 as 10 + 6 and consider the problem as $8 \times 16 = 8 \times (10 + 6) = (8 \times 10) + (8 \times 6) = 80 + 48 = 128$. A geometric area model is shown at the right; this method should also be taught **and** assessed.



The previous paragraph is only part of the story—only "left" distribution has been shown. "Right" distribution, where the factor being "distributed" is written to the right of the group, is equally as useful. That is, $16 \times 8 = (10 + 6) \times 8 = (10 \times 8) + (6 \times 8)$. Students must see the property presented in both forms. This leads to increased success in first-year algebra courses where right-distribution tends to cause students a fair degree of consternation, chiefly because they have never seen it before! Additionally, the distributive property can be generalized to any number of addends in the group instead of just two. There is no reason why students couldn't think of 8×316 as
$8 \times (300 + 10 + 6) = (8 \times 300) + (8 \times 10) + (8 \times 6) = 2400 + 80 + 48 = 2528$.

Multiplication is also distributive over subtraction. This should come as no surprise to us because we know subtraction is really addition of an opposite. (See *Take It to the MAT*, January 7, 2002.) We might think of 8×16 differently now, perhaps as $8 \times (20 - 4) = (8 \times 20) - (8 \times 4) = 160 - 32 = 128$.

It is important that students be flexible in using the distributive property with both mental and pencil-paper calculations. For example, when posed with 4×897, the natural inclination is to think $4 \times 897 = 4 \times (800 + 90 + 7)$. While it's not overly difficult to add the partial products of 3200, 360, and 28, it may be easier to think of it as $4 \times 897 = 4 \times (900 - 3)$, subtracting 12 from 3600.

Lastly, the distributive property can be used more than once in a particular problem. If we were asked to find the product of 23×45, we could think of this as $23 \times (40 + 5)$ which is $(23 \times 40) + (23 \times 5)$. But, isn't $23 \times 40 = (20 + 3) \times 40 = (20 \times 40) + (3 \times 40)$? And isn't $23 \times 5 = (20 + 3) \times 5 = (20 \times 5) + (3 \times 5)$? Therefore,
$23 \times 45 = (20 \times 40) + (3 \times 40) + (20 \times 5) + (3 \times 5) = 800 + 120 + 100 + 15 = 1035$.
Hey, that's what we do when we use a multiplication algorithm!
(See the box to the right. A geometric model is also provided.)

23		
×45		$\overset{1}{\cancel{2}}$
15		23
100	or	×45
120		115
800		920
1035		1035