TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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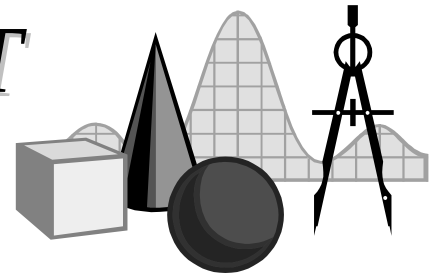

Divisibility Rules for Whole Numbers	
3 —	Sum of digits is a multiple of 3
6 —	Number is divisible by both 2 and 3
7 —	Difference between twice last digit and number formed by previous digits is a multiple of 7
9 —	Sum of digits is a multiple of 9
11 —	Difference of sums of alternating digits is a multiple of 11

Once students have mastered the divisibility rules for 1, 2, 4, 5, 8, and 10, they can move on to other rules. In this issue of *Take It to the MAT*, we will discuss the order in which the rules should be taught and some cautions about divisibility rules in general.

Following 5, 10, 2, 4, and 8, the next rule that can be taught is 3. The rule for 3 is easy—add up the digits and if the sum is a multiple of 3, so is the original number. For example, 288 is divisible by three because the sum of its digits, 2 + 8 + 8 = 18, is also divisible by three.

The proof of this is within the reach of most Algebra I students. It is left out here due to space concerns and may be addressed in a future issue of *Take It to the MAT*.

After 3, the next rule learned is for 9, mainly because it functions in the same manner as 3. In the previous example, 288 is divisible by 9 because the sum of its digits is also divisible by 9. The proof of why it works is also similar to that of 3.

Now that the law for divisibility by 3 is developed, we can proceed to 6. The rule for divisibility by 6 is simply that the number must be divisible by both 2 and by 3. If a number is divisible by 6, it has a factor of 6. That factor can be further broken down into factors of 2 and 3. Since 288 is divisible by 2 and by 3, it is also divisible by 6.

The method for divisibility by 6 can be extended to many other numbers. A number is divisible by 15 if it is divisible by 3 and by 5; it is divisible by 12 if divisible by both 3 and 4. This method only works, however, if the factors tested are *relatively prime*, that is, they only have a common factor of 1. For example, 60 is divisible by both 3 and 12, but it is not divisible by 36; 36 is divisible by both 2 and 4, but not 8.

Lastly, there are rules for 7 and 11. For 7, take the last digit, double it, then subtract it from the "number" formed by the removal of the last digit. If that difference is divisible by 7, so is the original number. For example, look at 2926. The last digit is 6, doubled is 12. Subtract that from 292. The result of 280 is divisible by 7, thus 2926 is also.

For 11, add alternating digits and then subtract those sums. If the difference is divisible by 11, so is the number. Again, look at 2926. The first and third digit sum to 4, the second and fourth add up to 15. Subtracting 4 from 15, we get 11, which is divisible by 11. Thus, 2926 is divisible by 11. (Note: 2926 is divisible by both 7 and 11—which are *relatively prime*—therefore it is also divisible by 77.)

There are rules for 13, 17, and just about any other number. But there comes a time when their utility is questionable or their complexity makes it more desirable to just divide the number the "old-fashioned" way.

Finally, teachers are reminded to *not* teach the rules in numerical order. Since many rules connect to other rules by their methods, it makes more sense to teach them in the order presented here.