TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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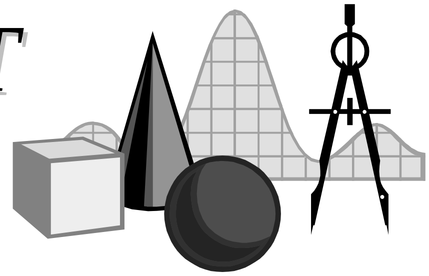

Often we call upon students to simplify fractions or find common factors. Tools that are helpful with these processes are divisibility rules. This edition of *Take It to the MAT* focuses on rules of divisibility, why they work, in what order they should be learned, and which ones should be skipped.

Just as it is inefficient, and sometimes counterproductive, to learn multiplication facts in order—0's, 1's, 2's, 3's, 4's, etc., up to 12's—it is also bad practice to teach divisibility rules in order. There are connections among the various multiplication facts, it makes sense to develop divisibility rules in a similar order.

Divisibility Rules for Whole Numbers

- 1 — Any number is divisible by 1
- 2 — Last digit is 0, 2, 4, 6, or 8
- 4 — Number formed by last two digits is a multiple of 4
- 5 — Last digit is 0 or 5
- 8 — Number formed by last three digits is a multiple of 8
- 10 — Last digit is 0

We start with the rules that students find easy—1, 2, 5, and 10. Most students learn the 1's, 2's, 5's, and 10's multiplication facts in a short time. Similarly, few students have difficulty understanding the 1's, 2's, 5's, and 10's divisibility rules and why they work.

Precursory knowledge to some of the divisibility rules is understanding that if two (or more) numbers are divisible by a given value, the sum (or difference) of numbers is also divisible by the value. For example, 27 and 39 are both divisible by 3. Twenty-seven is 9 threes and thirty-nine is 13 threes. If we add 27 and 39, we are adding 9 threes and 13 threes, so we get 22 threes or 66.

What's next? Divisibility by 4 is a good place. Seeing why the rule works is not too difficult for most students either. If the number is one or two digits, the answer is easy. If the number is three or more digits, think about the number as some number of hundreds plus the tens and ones.

For example, is 1348 divisible by 4? Think of 1348 as 1300 + 48. Any multiple of 100 is divisible by 4 because 100 is divisible by 4, so 1300 is divisible by 4. We know that 48 is divisible by 4—it is 12×4. So 1348 is divisible by 4, since the number formed by the last two digits is divisible by 4. We need not deal with the hundreds each time; only the rule for the last two-digits need be applied.

Divisibility by 8 is the logical next step, but knowing three-digit multiples of 8 is not as easy as two-digit multiples of 4. So, a suggested alternate strategy would be to take the number formed by the last three digits and divide it by two—something students should be able to do mentally. If the result is now divisible by 4, the original number is divisible by 8 since $2 \times 4 = 8$.

For example, is 32,378 divisible by 8? According to the rule, we need only check if 378 is divisible by 8 since any multiple of 1,000 already is. As we can't immediately see if 378 is a multiple of 8, we can divide it by 2. Half of 378 is 189, which, when we look at 89, is definitely *not* a multiple of 4, so 32,378 is not a multiple of 8. Had the original number been 32,376, half of 376 is 188, which is a multiple of 4—since 88 obviously is. So, 32,376 is a multiple of 8.

We could also use this halving technique with 4. If half of the two-digit number is divisible by 2, then the two-digit number is divisible by 4. For instance, since 46 is even and is half of 92, 92 is divisible by 4.

Next time, divisibility for 3, 6, 7, 9, and 11.