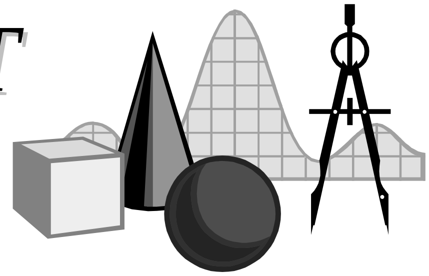
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
Regional Professional Development Program
November 27, 2000 — Elementary Edition



This issue's *Take It to the MAT* topic is by request from a teacher at J. T. McWilliams ES.

The question is how the procedure *casting out nines* works. *Casting out nines* is a method to check the accuracy of a sum, difference, or product. Sometimes referred to as the *nines test*, casting out nines works as follows:

> *The remainder of division by nine of a sum (a difference, a product) is equal to the sum (the difference, the product) of the individual remainders.*

For example, let us check 319 + 234 = 553. When 319 is divided by 9, the remainder is 4; 234 ÷ 9 has a remainder of 0. The sum 553 has a remainder of 4. The check is that the sum of the addends' remainders is equal to the sum's remainder. In this case our arithmetic is likely accurate since 4 + 0 = 4. It takes only a moment to show that the same is true for 319 – 234.

Problem		Remainder
319	⇨	4
+234	⇨	+0
553	⇨	4 ✓

What about multiplication? Is 319 × 234 = 74,546? Let's check. When divided by nine, 319 and 234 have remainders of 4 and 0 respectively. By the rule above, the remainder of the product should be 0 since the product of 4 × 0 = 0. When 74,546 is divided by 9 the remainder is 8, not 0—we must have made a mistake. Double-checking our arithmetic, 319 × 234 = 74,656; 74,656 has a remainder of 0 when divided by 9.

Problem		Remainder
319	⇨	4
×234	⇨	×0
74,546	⇨	8 **?**

Notice that in the first example we said it is *likely* that our arithmetic is correct. Could our arithmetic be wrong yet still get the remainders right? Let's imagine that we were very poor with arithmetic and found the product of 319 and 234 to be 62,316. While the product is incorrect, the remainders work out. But, it is *unlikely* that we (or a student) would be so far off for this to occur.

Problem		Remainder
319	⇨	4
×234	⇨	×0
62,316	⇨	0 ✓

While the nines test is valuable, the thought of having to divide a five-digit product by nine—or even a three-digit factor for that matter— to determine its remainder is troubling because the division process is prone to errors. Another neat property of the number nine is that the remainder of a number when divided by nine is equal to the remainder when the sum of the number's digits is divided by nine. This provides a nice shortcut to finding remainders. If 553 is divided by 9, the remainder is 4. The sum of 553's digits is 13; when divided by 9, 13 has a remainder of 4. Check it out yourself with each of the examples above.

One can prove with some simple algebra that casting out nines and the sum of digits shortcut work. The proof may be beyond the comprehension of elementary students and space limitations prevent its presentation here, but it may be addressed it in future issues. The reader is encouraged to individually investigate this in the interim.