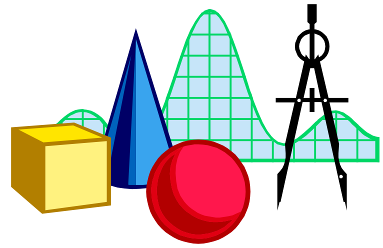
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
December 3, 2001 — Elementary School Edition



As students progress through their mathematical education in the elementary and middle grades, the groundwork is laid for algebra, the gateway to higher mathematics and science. One of the cornerstones of the algebraic foundation is an understanding of and proficiency with the properties of real numbers. There are many properties, two of which refer to operations and are addressed in this issue of *Take It to the MAT*: *associative*, and *commutative*.

Most properties we use are for real numbers. Real numbers include whole numbers, integers, rational numbers (numbers that can be written as a fraction), and irrational numbers (numbers that cannot be written as a fraction, such as $\sqrt{2}$).

Addition and multiplication are *associative* in the real numbers. That is, the sum or product of a set of three numbers is unique regardless of how we group them with grouping symbols. We usually see the associative property defined in symbols as $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$, followed with examples such as $(8 + 7) + 3 = 8 + (7 + 3)$ and $(9 \times 4) \times 5 = 9 \times (4 \times 5)$. This property is very powerful in that students who are accustomed to doing arithmetic from left to right, may now perform operations in a different order to simplify calculations. In the examples shown, grouping to make a 10 (in the addition case) or the product of 20 (in multiplication) makes the calculation easier.

Addition and multiplication are *commutative* in the real numbers. That is, the sum or product of two values is unique regardless of the order in which they are written. We define the commutative property in symbols as $a + b = b + a$ and $a \times b = b \times a$, followed with examples such as $8 + 7 = 7 + 8$ and $9 \times 4 = 4 \times 9$. The benefit is that students don't get hung up on the order of numbers in an operation.

While the associative property is only defined for three numbers and the commutative property only for two, they can (through algebraic proof not done here) be generalized to any size set of numbers. In effect, we can add or multiply any numbers in any order that we wish. We need not operate from left to right. For example, if presented with 19 + 43 + 51 + 15 + 67, we could add 19 and 51 first (70), then 43 and 67 (110). Now, we have 70 + 110 + 15, which is 180 + 15 = 195.

$$
\begin{aligned}
&19+43+51+15+67\\
&=(((19+43)+51)+15)+67\\
&=((19+(43+51))+15)+67\\
&=((19+(51+43))+15)+67\\
&=(((19+51)+43)+15)+67\\
&=((19+51)+(43+15))+67\\
&=((19+51)+(15+43))+67\\
&=(19+51)+(15+(43+67))\\
&=(19+51)+((43+67)+15)\\
&=((19+51)+(43+67))+15\\
&=(70+110)+15\\
&=180+15\\
&=195
\end{aligned}
$$

In the above example, we employed the associative and commutative properties rather informally to regroup and reorder numbers that are compatible and make multiples of ten. This eased our burden somewhat over adding them vertically with the addition algorithm or adding them mentally from left to right. (The formal application of the properties is shown to the right. This is *NOT* what we should expect kids to do! It is provided as a point of interest.)

The point is that students should know the terms *associative property* and *commutative property*, know how they pertain to the operations of addition and multiplication, and be fluent in their use.