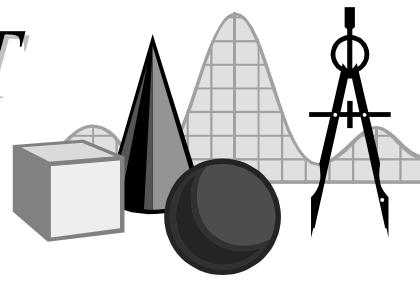


TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
May 8, 2000 — Elementary Edition

Mathematics is a symbolic language. We use funny-looking characters to describe mathematical operations, relationships, and special numbers. In this edition of *Take It to the MAT*, we will feed on a smorgasbord of mathematical topics using symbols and notation involving equality. Each course will be a discussion of how notation is used appropriately and how it should not be used.

String (beans) of equals. Consider this problem: Calculate $5 \times 2 + 3$. Seems simple enough; the answer is 13. But very often we get sloppy and write $5 \times 2 = 10 + 3 = 13$ as we say, “Five times two equals ten; ten plus three equals 13.” What we have said is true, but what we have written is not! Whenever a number sentence is written, all “sides” of the equals signs must be equivalent. For example, $2 + 2 + 2 = 2 \times 3 = 6$ is correct because $2 + 2 + 2 = 6$, $2 + 2 + 2 = 2 \times 3$, and $2 \times 3 = 6$. But what was written above was not correct because $5 \times 2 \neq 10 + 3$, nor is $5 \times 2 = 13$.

Cooking up values for pi. The Greek letter π (pi) is the traditional symbol to represent the ratio between the circumference of any circle and its radius. The value of pi used by the ancient Egyptians was 3.16. Archimedes calculated π to be between $3\frac{10}{71}$ (or $\frac{223}{71}$) and $3\frac{1}{7}$ (or $\frac{22}{7}$). Many texts suggest to use 3.14 or 3.1416 for pi. Why so many different values? Because pi is an irrational number; it cannot be written as the ratio of two integers. Expressed in decimal form, pi never terminates or repeats like some other decimals ($\pi = 3.1415926535\dots$). Pi has been calculated to a billion-plus decimal places with no end or repetition of digits.

Since we cannot use the actual value of pi, we use an *approximation*: $\pi \approx \frac{22}{7}$ or $\pi \approx 3.14$. Note that little squiggly symbol (\approx); it means *approximately*. We can never give an exact value of pi, so sometimes we use an approximation. Thus, we must use the proper notation, \approx , when writing what value is used for pi, or when doing calculations with pi.

Add $\frac{1}{3}$ rounded teaspoon of Very often there is a need to change a number expressed as a fraction to a decimal form. For example: $\frac{1}{2} = 0.5$, $\frac{3}{8} = 0.375$, or $\frac{5}{9} \approx 0.556$. Now, here's a question: did you see it? Look at that last conversion. Notice the use of the approximate symbol as discussed in the previous paragraph. Why is it there? Because $\frac{5}{9} = 0.555555\dots$ The decimal form of $\frac{5}{9}$ never terminates, repeating the digit 5 forever. Unless we want our hand to cramp up writing an infinite number of decimal places, we'll have to round it off somewhere. No matter whether we round it to one place or ten, we'll still have an approximation.

We may wish to round off the decimal form of some fractions where the decimal does terminate, like $\frac{1}{128} = 0.0078125$. If we don't write the entire decimal, we must use the approximation symbol, as in $\frac{1}{128} \approx 0.0078$.

Clearing the table. Many times students are left with false impressions when equals signs are used carelessly. It is important that mathematical notation be used with accuracy and precision, no matter the circumstances.